

## Method of Variation of parameters

$$y'' + p(x)y' + q(x)y = r(x) \quad (*)$$

$p(x)$ ,  $q(x)$ , and  $r(x)$  are continuous functions on an interval  $I$ .

Let  $y_1$  and  $y_2$  be two linearly independent (on  $I$ ) solutions of the homogeneous equation. Then there is a particular solution of the (\*) of the form  $y_p = uy_1 + vy_2$ , where

$$\begin{aligned} u'y_1 + v'y_2 &= 0 \\ u'y'_1 + v'y'_2 &= r \end{aligned}$$

Solving the system, we get the formula

$$y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx,$$

where  $W = W(y_1, y_2) = y_1 y'_2 - y'_1 y_2$  is the Wronski determinant of  $y_1$  and  $y_2$ .