Elementary row operations

1. Multiply a row of the matrix by a non-zero constant.

2. Interchange two rows of the matrix.

3. Add a multiple of one row to another one.

Row-equivalent matrices

Two matrices are called row-equivalent if one can be obtained from the other by a sequence of elementary row operations.

Theorem If augmented matrices of two linear systems are row-equivalent, then the two systems have the same set of solutions.

Echelon Matrices

A matrix is called an echelon matrix if

(i) each row consisting of zeros lies beneath every row that contains non-zero elements,

(ii) in each row that contains a nonzero element, the first nonzero element lies strictly to the right of the first nonzero element in the preceding row.

The first nonzero element in each row is called a leading entry of the matrix.

Suppose that the augmented matrix of a linear system is in echelon form. Then each (but the last) column of the matrix corresponds to a variable.

The variables corresponding to the columns containing leading entries are called leading variables.
Other variables are called free variables.

Solution of Linear systems by Gauss Elimination

1. Apply elementary row operations to the augmented coefficient matrix

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{array}\right]$$

to obtain a row-equivalent echelon matrix.

2. If the resulting matrix contains a row

$$0 \; 0 \; \dots \; 0 \; |b$$

with non-zero b then the system has no solution.

3. Otherwise we can solve the linear system in echelon form by back substitution. Set each free variable equal to an arbitrary parameter and solve the equation for each leading variable starting from the last equation and going upward through the system.