Inverses of Matrices

Definition A square matrix A is called invertible if there exists a matrix B such that $AB=BA=I$, where I is the identity matrix.

Theorem 1. If A is an invertible matrix then there exists precisely one matrix B such that $AB=BA=I$. It is called the inverse of A and is denoted by A^{-1}.

2. If A is invertible then A^{-1} is invertible and $(A^{-1})^{-1} = A$.

3. If A and B are invertible matrices of the same size then AB is invertible and

$$(AB)^{-1} = B^{-1}A^{-1}.$$

Theorem Let A be a square matrix. The following conditions are equivalent:

1) $Ax=0$ has only the trivial solution $x=0$,
2) A is row-equivalent to the identity matrix,
3) for any b the equation $Ax=b$ has presicely one solution,
4) for any b the system $Ax=b$ is consistent,
5) there exists a matrix B such that $AB=I$,
6) A is invertible.