Definition of Vector Space

Let *V* be a set of elements in which operations of addition and multiplication by scalars are defined. *V* is called a vector space if the following conditions are satisfied:

(a) $\boldsymbol{u}+\boldsymbol{v}=\boldsymbol{v}+\boldsymbol{u}$

(b) $\boldsymbol{u}+(\boldsymbol{v}+\boldsymbol{w})=(\boldsymbol{u}+\boldsymbol{v})+\boldsymbol{w}$

(c) $\boldsymbol{u}+\boldsymbol{0}=\boldsymbol{0}+\boldsymbol{u}=\boldsymbol{u}$

(d) $\boldsymbol{u}+(-\boldsymbol{u})=(-\boldsymbol{u})+\boldsymbol{u}=\boldsymbol{0}$

(e) $a(\boldsymbol{u}+\boldsymbol{v})=a\boldsymbol{u}+a\boldsymbol{v}$

(f) $(a+b)\boldsymbol{u}=a\boldsymbol{u}+b\boldsymbol{u}$

(g) $a(b\boldsymbol{u})=(ab)\boldsymbol{u}$

(h) $1\boldsymbol{u}=\boldsymbol{u}$

For any elements $\boldsymbol{u}$,$\boldsymbol{v}$, and $\boldsymbol{w}$ in *V* and any scalars *a* and *b*.

Subspace

Defenition Let *W* be a nonempty subset of a vector space *V*. Then *W* is called a subspace of *V* provided that *W* is itself a vector space with operations as in *V*.

Theorem A nonempty subset *W* of a vector space is a subspace if and only if for any vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in *W* and any scalar *c* vectors $\boldsymbol{u}+\boldsymbol{v}$ and $c\boldsymbol{u}$ are also in *W*.