Linear Combinations

Definition A vector $\boldsymbol{w}$ is called a linear combination of vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k$ if there exist constants $c_1, c_2, \ldots, c_k$ such that $\boldsymbol{w} = c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + \ldots + c_k\boldsymbol{v}_k$.

Theorem If $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k$ are vectors in a vector space then the set W of all linear combinations of $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k$ is a subspace of the vector space.

W is called the span of the set $S=\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$, $W=\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$.

Linear Independence

Defenition Vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k$ in a vector space V are said to be linearly independent if the equation $c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + \ldots + c_k\boldsymbol{v}_k = 0$ has only the trivial solution.

Vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k$ are linearly dependent if there exist constants $c_1, c_2, \ldots, c_k$ not all zeros such that $c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + \ldots + c_k\boldsymbol{v}_k = 0$.