Bases of Vector Spaces

Definition A finite set S of vectors in a vector space V is said to be a **basis** for V provided that

(a) the vectors in S are linearly independent

(b) the vectors in S span V.

Theorem Any two bases for a vector space consist of the same number of vectors.
This number is called the **dimension** of the vector space.

Theorem Let V be an n-dimensional vector space and let S be a subset of V. Then

(a) If S is linearly independent and consists of n vectors then S is a basis for V;

(b) If S spans V and consists of n vectors then S is a basis for V;

(c) If S is linearly independent then S is contained in a basis for V;

(d) If S spans V then S contains a basis for V.

The Solution Space

Consider a homogeneous system of linear equations $A\mathbf{x}=0$ (here A is an m by n matrix). The space of all solutions is a subspace of R^n, it is called the solution space and is denoted by Null(A).