Dot product in R^n

The dot product of two vectors $\boldsymbol{u}=(u_1,u_2,\ldots,u_n)$ and $\boldsymbol{v}=(v_1,v_2,\ldots,v_n)$ in R^n is defined to be $\boldsymbol{u}\cdot\boldsymbol{v}=u_1v_1+u_2v_2+\ldots+u_nv_n$.

Properties

1. $\boldsymbol{u}\cdot\boldsymbol{v} = \boldsymbol{v}\cdot\boldsymbol{u}$
2. $\boldsymbol{u}\cdot(\boldsymbol{v}+\boldsymbol{w}) = \boldsymbol{u}\cdot\boldsymbol{v}+ \boldsymbol{u}\cdot\boldsymbol{w}$
3. $(c\boldsymbol{u})\cdot\boldsymbol{v} = c(\boldsymbol{u}\cdot\boldsymbol{v})$
4. $\boldsymbol{u}\cdot\boldsymbol{u} \geq 0$; $\boldsymbol{u}\cdot\boldsymbol{u}=0$ if and only if $u=0$

Definition The length of the vector $\boldsymbol{u}=(u_1,u_2,\ldots,u_n)$ in R^n is defined by $|\boldsymbol{u}|=(u_1^2+u_2^2+\ldots+u_n^2)^{1/2}$. The distance between two points (vectors) $\boldsymbol{u}$ and $\boldsymbol{v}$ in R^n is given by $d(\boldsymbol{u},\boldsymbol{v})=|\boldsymbol{u}-\boldsymbol{v}|$.

Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be two vectors in R^n, then the following inequalities are satisfied

Cauchy-Schwarz Inequality $|\boldsymbol{u}\cdot\boldsymbol{v}| \leq |\boldsymbol{u}||\boldsymbol{v}|$

Triangle Inequality $|\boldsymbol{u}+\boldsymbol{v}| \leq |\boldsymbol{u}|+|\boldsymbol{v}|$

Definition Suppose that $\boldsymbol{u}$ and $\boldsymbol{v}$ are two nonzero vectors in R^n. Then the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is a unique number θ, $0\leq\theta\leq\pi$, such that $\boldsymbol{u}\cdot\boldsymbol{v}= |\boldsymbol{u}||\boldsymbol{v}|\cos\theta$.