Orthogonal Vectors in R^n

Definition Two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in R^n are called orthogonal if $\boldsymbol{u}\cdot\boldsymbol{v}=0$.

Pythagorean's formula If $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal then $|\boldsymbol{u}+\boldsymbol{v}|^2=|\boldsymbol{u}|^2+|\boldsymbol{v}|^2$.

Theorem If nonzero vectors $\boldsymbol{v}_1,\boldsymbol{v}_2,\ldots,\boldsymbol{v}_k$ are mutually orthogonal they are linearly independent.

Orthogonal Complements

Definition A vector $\boldsymbol{u}$ is orthogonal to a subspace V of R^n provided that $\boldsymbol{u}$ is orthogonal to every vector in V. The set of all those vectors in R^n that are orthogonal to the subspace V is called the orthogonal complement of V.
We denote the orthogonal complement of V by $V^{\perp}$.

Properties
1. $V^{\perp}$ is a subspace of R^n,
2. the only vector that lies in both V and $V^{\perp}$ is zero vector,
3. $(V^{\perp})^{\perp}=V$.

$$\dim V + \dim V^{\perp} = n$$