# Eigenvalues and Eigenvectors

**Definition** Let $A$ be an $n$ by $n$ matrix. A number $\lambda$ is called an eigenvalue of $A$ if there exists a nonzero vector $\boldsymbol{v}$ such that $A\boldsymbol{v}=\lambda\boldsymbol{v}$.

**Definition** A nonzero vector $\boldsymbol{v}$ such that $A\boldsymbol{v}=\lambda\boldsymbol{v}$ is called an eigenvector of $A$ corresponding to the eigenvalue $\lambda$.

**Theorem** A number $\lambda$ is an eigenvalue of $A$ if and only if $\det(A-\lambda I) = 0$ .

The characteristic equation $\det(A-\lambda I) = 0$ is a polynomial equation in $\lambda$ of degree $n$.

To find the eigenvalues and eigenvectors of a square matrix

1. Solve the characteristic equation $\det(A-\lambda I) = 0$.
2. For each eigenvalue $\lambda$ solve the linear system $(A-\lambda I)\boldsymbol{v}=0$ and find the eigenvectors corresponding to $\lambda$.

If we fix an eigenvalue $\lambda$ then the solutions of the system $(A-\lambda I)\boldsymbol{v}=0$ form a subspace of $R^n$ that is called the eigenspace of $A$ associated with $\lambda$.