Diagonalization of Matrices

Definition Square matrices A and B are called **similar** if there exists an invertible matrix P such that $B=P^{-1}AP$.

Definition A square matrix is called **diagonalizable** if it is similar to a diagonal matrix.

Theorem An n by n matrix is diagonalizable if and only if it has n linearly independent eigenvectors.

If matrix A is diagonalizable, $A=PDP^{-1}$, then D is a diagonal matrix with eigenvalues of A on the diagonal, it is called the eigenvalue matrix, columns of P are linearly independent eigenvectors of A and P is called the eigenvector matrix.

Theorem If $v_1, v_2, \ldots, v_k$ are eigenvectors of A corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$, then they are linearly independent.

Corollary If an n by n matrix A has n distinct eigenvalues it is diagonalizable.

If A is a diagonalizable matrix, $A=PDP^{-1}$, then we can calculate the powers of A using the formula

$$A^m=PD^mP^{-1}$$