Cramer's Rule

Theorem Consider a linear system Ax=b and suppose that det A $\neq$ 0. Then the system has presicely one solution given by

$$x_i = \frac{1}{\det A} \begin{vmatrix} a_{11} & \ldots & b_1 & \ldots & a_{1n} \\ a_{12} & \ldots & b_2 & \ldots & a_{2n} \\ . & . & . & . & . \\ a_{n1} & \ldots & b_n & \ldots & a_{nn} \end{vmatrix},$$

where b replaces the ith column of A.

The inverse and the adjoint matrix

Let A be a square matrix, A_{ij} denote the ijth cofactor of A. Then the adjoint matrix of A is defined by adj $A=[\, A_{ij}]^T$.

Theorem If A is an invertible matrix then

$$A^{-1} = \frac{\text{adj } A}{|\, A \,|}.$$