First-Order Systems: General Theory

$$y_1' = f_1(t, y_1, y_2, \ldots, y_n)$$
$$y_2' = f_2(t, y_1, y_2, \ldots, y_n)$$
$$\cdots$$
$$y_n' = f_n(t, y_1, y_2, \ldots, y_n)$$

The above system can be written as a vector equation $\boldsymbol{y}' = \boldsymbol{f}(t, \boldsymbol{y})$, where $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)^T$ and $\boldsymbol{f} = (f_1, f_2, \ldots, f_n)^T$.

Initial value problem $\boldsymbol{y}' = \boldsymbol{f}(t, \boldsymbol{y})$, $\boldsymbol{y}(t_0) = \boldsymbol{y}_0$.

Linear Systems are systems of the form

$$y_1' = a_{11}(t)\, y_1 + a_{12}(t) y_2 + \ldots + a_{1n}(t) y_n + g_1(t)$$
$$y_2' = a_{21}(t)\, y_1 + a_{22}(t) y_2 + \ldots + a_{2n}(t) y_n + g_2(t)$$
$$\cdots$$
$$y_n' = a_{n1}(t)\, y_1 + a_{n2}(t) y_2 + \ldots + a_{nn}(t) y_n + g_n(t)$$

it can be written as a vector equation $\boldsymbol{y}' = A\boldsymbol{y} + \boldsymbol{g}$.
The system is called homogeneous if g=0.

Superposition Principle for Homogeneous Systems

If $\boldsymbol{y}^{(1)}$ and $\boldsymbol{y}^{(2)}$ are solutions of the homogeneous system $\boldsymbol{y}' = A\boldsymbol{y}$ then $c_1\boldsymbol{y}^{(1)} + c_2\boldsymbol{y}^{(2)}$ is also a solution.