

Homogeneous Systems with Constant Coefficients

Homogeneous linear systems with constant coefficients are systems of the form $\mathbf{y}' = A\mathbf{y}$, where A is a constant n by n matrix.

Theorem If the matrix A has n linearly independent eigenvectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$, corresponding to eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ then the general solution to the above system is given by the formula

$$\mathbf{y} = c_1\mathbf{v}_1\exp(\lambda_1 t) + c_2\mathbf{v}_2\exp(\lambda_2 t) + \dots + c_n\mathbf{v}_n\exp(\lambda_n t).$$

To solve a nonhomogeneous linear system $\mathbf{y}' = A\mathbf{y} + \mathbf{g}$ we first find the general solution of the homogeneous system and then add one particular solution of nonhomogeneous.