Norwegian University of Science and Technology
Departement of Mathematical Sciences

TMA4110
Calculus 3
Autumn 2010

Exercise set 8 – Week 40

Edwards & Penney, section 2.4

5,15,23

Edwards & Penney, section 4.1

3,8,19,30

Exam problems

(`SIF5009 december 2000`)

7 Let A be an $n \times n$-matrix and let $\mathbf{x}$ be an n-vector such that $A^3\mathbf{x} = \mathbf{0}$, while $A^2\mathbf{x} \neq \mathbf{0}$. Show that the vectors $\mathbf{x}$, $A\mathbf{x}$ and $A^2\mathbf{x}$ are linearly independent.

(`SIF5010 august 2003`)

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Given the matrix $A = \begin{bmatrix} 1 & 0 & 0 & -\alpha \\ 0 & 0 & \alpha & 1 \\ 0 & \alpha & 1 & \alpha \\ \alpha & 1 & \alpha & 0 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} \alpha \\ 0 \\ \alpha \\ 1+\alpha \end{bmatrix}$ where $\alpha \in \mathbb{R}$.

a) Compute $\det(A)$ and decide for which values of α the matrix A is invertible.

b) For which values of α does the system of equations $A\mathbf{x} = \mathbf{b}$ have exactly one solution, infinitely many solutions or no solution, respectively?

Multiple-choice questions

1 Suppose that A is a 5×7–matrix. What can you say about the number of free variables, k, for the system $A\mathbf{x} = \mathbf{0}$?

A: $k \leq 2$ **B:** $k = 2$ **C:** $2 \leq k \leq 7$ **D:** $k \leq 7$

2 Compute the rank r for the 3×4–matrix

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 3 & 1 \end{bmatrix}.$$

A: $r = 1$ **B:** $r = 2$ **C:** $r = 3$ **D:** $r = 4$