Norwegian University of Science and Technology
Departement of Mathematical Sciences

Exercise set 11 – Week 45

Edwards & Penney, section 5.2

3,13,21,25

Edwards & Penney, section 5.4

2,11,25

Exam problems

A-40 **a)** Use the Gram-Schmidt orthogonalization algorithm to find an orthogonal basis for the subspace $V \subseteq \mathbb{R}^4$ spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix}1\\1\\0\\1\end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix}2\\0\\1\\1\end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix}1\\3\\5\\-1\end{bmatrix}.$$

b) Find the orthogonal projection of $\mathbf{b} = \begin{bmatrix}0\\4\\7\\5\end{bmatrix}$ into the subspace V.

Des. 07, oppg. 6 **a)** Find a basis for the solution space of the homogeneous system

$$\begin{aligned} x_1 - 2x_2 + x_3 \qquad &= 0 \\ x_1 \qquad - x_3 + 2x_4 &= 0. \end{aligned}$$

b) Find the orthogonal projection of the vector $(1, 2, -3, 1)$ into the subspace $V = \text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ in $\mathbb{R}^4$ where $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal vectors given by

$$\mathbf{v}_1 = (1, -2, 1, 0), \qquad \mathbf{v}_2 = (1, 0, -1, 2)$$

c) Find vectors $\mathbf{v}_3$ and $\mathbf{v}_4$ such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is an orthogonal basis for $\mathbb{R}^4$.

Multiple-choice questions

1 Suppose that A is a 4×3-matrix. What is rank A?

A: at most 3 **B:** 3 **C:** at least 3 **D:** 4

Which of the following alternatives is the least-squares solution $(\overline{x}, \overline{y})$ of the system

$$-x + y = 5, \quad -x + 2y = 0, \quad -3x + y = -5?$$

A: $(2, 3/2)$ **B:** $(1, 1)$ **C:** $(3/2, 3/2)$ **D:** $(2, 2)$