NTNU
Norwegian University of Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4110 Calculus 3**

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Examination date: 4 December 2018

Examination time (from–to): 09:00–13:00

Permitted examination support material: No printed or hand-written support material is allowed. A specific basic calculator is allowed. (Casio fx-82ES PLUS, Casio fx-82EX, Citizen SR-270X, Citizen SR-270X College, Hewlett Packard HP30S)

Other information:
The exam consists of ten problems. Each of these carries the same weight towards the grade. Give reasons for all answers.

Language: English

Number of pages: 2

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave	
Originalen er:	
1-sidig ☐	**2-sidig** ☒
sort/hvit ☒	**farger** ☐
skal ha flervalgskjema ☐	

Date　　Signature

Problem 1 Find all solutions of the following system of equations:

$$\begin{cases} x - y + 2z = 28 \\ -2x + 5y - 4z = -20 \\ -x + y - z = -10 \end{cases}$$

Problem 2 Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -3 \\ -6 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ -9 \\ 3 \end{bmatrix}$$

Are the vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ linearly independent? Is $\mathbf{b}$ a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$?

Problem 3 Find the general solution of the system

$$\begin{cases} y_1' = 7y_1 - 2y_2 \\ y_2' = 2y_1 + 2y_2 \end{cases}$$

and sketch the phase plane plot.

Problem 4 Consider the three points

$$\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

in $\mathbb{R}^2$.

Find the polynomial $p(x) = ax^2 + bx + c$ of degree two that goes through all these points.

Use the method of least squares to find the polynomial $q(x) = dx + e$ of degree one that best fits the three points.

Draw the graphs of p and q.

Problem 5 Let A be the following matrix:

$$A = \begin{bmatrix} 9 & -3 \\ -3 & 2 \end{bmatrix}$$

Find all 2×2-matrices X that are solutions of the equation $AX = XA$.

Problem 6 Find an orthogonal basis for the subspace of $\mathbb{R}^4$ spanned by these vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

Problem 7 Let R be the following matrix:

$$R = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$$

Compute R^{42}.

Problem 8 Let A be the following complex matrix:

$$A = \begin{bmatrix} 2 & i & 5-3i \\ 4 & 2i & 10+2i \\ 2i & -1 & 4+6i \end{bmatrix}$$

First: Find a basis for $\operatorname{Null} A$ and a basis for $\operatorname{Col} A$.

Then: Find all vectors $\mathbf{v}$ in $\mathbb{C}^3$ such that $A\mathbf{v} = \mathbf{0}$ and $\|\mathbf{v}\| = 1$.

Problem 9 Recall that we write $\mathcal{M}_2$ for the vector space consisting of all real 2×2-matrices. Define a function $T \colon \mathcal{M}_2 \to \mathcal{M}_2$ by

$$T(M) = M - M^\top.$$

Show that T is a linear transformation, and find $\ker T$ and $\operatorname{im} T$.

Problem 10 Let A be an $m \times n$-matrix with linearly independent columns, and let B and C be $n \times p$-matrices. Show that if $AB = AC$, then $B = C$.