1] In a metropolitan area with constant total population, 7 million people lives in the city centre and 5 million people lives in the suburbs. Every year 20 % of the inhabitants of the centre moves to the suburbs (80 % stays in the centre), while 10 % of the people living in the suburbs moves to the centre (90 % stays in the suburbs).

In the long term, what is the distribution of inhabitants between the centre and the suburbs?

**Hint:** The matrix

\[
A = \begin{bmatrix}
0.8 & 0.1 \\
0.2 & 0.9
\end{bmatrix}
\]

can be diagonalized as

\[A = PDP^{-1},\]

with

\[
P = \begin{bmatrix}
1 & -1 \\
2 & 1
\end{bmatrix}, \quad D = \begin{bmatrix}
1 & 0 \\
0 & 0.7
\end{bmatrix}.
\]

Furthermore, it holds that

\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix}
d & -b \\
-c & a
\end{bmatrix}.
\]

2] In this exercise we will study the symmetric matrix

\[
A = \begin{bmatrix}
2 & 0 & -1 \\
0 & 4 & 0 \\
-1 & 0 & 2
\end{bmatrix}.
\]

a) Find the characteristic polynomial of \(A\) (on factorised form)

(Hint: Choose wisely which column/row to start with.)

b) Find all the eigenvalues of \(A\) and the corresponding eigenvectors.

c) Find the orthogonal diagonalization of \(A\).

d) Find the matrix exponential \(e^A\) by using the orthogonal diagonalization of \(A\).