Norwegian University of Science and Technology
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Introductory course in linear algebra and differential equations Autumn 2019

Solutions to exercise set 1

$\boxed{1}$ **a)** This is not a linear transformation, as

$$S(0,1) + S(1,0) = (0,2) \neq (1,2) = S(1,1).$$

(Many ways lead to Rome in this case.)

b) This is a linear transformation, with matrix

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

c) Not a linear transformation, as $K(0) \neq 0$.
(Why must 0 map to 0 by a linear transformation?)

d) This is a linear transformation, with matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

e) Not a linear transformation as

$$\sin(\pi) = 0 \neq 2 = \sin(\pi/2) + \sin(\pi/2)$$

$\boxed{2}$ The linear transformation L is reflecting the plane around the x_2 axis (make a drawing!).

$\boxed{3}$ **a)**

$$\begin{bmatrix} 0 & 6 & 2 \\ 9 & -6 & -2 \\ 2 & -1 & 12 \end{bmatrix}$$

b)

$$\begin{bmatrix} 2 & -3 \\ 4 & -12 \end{bmatrix}$$

c)

$$\begin{bmatrix} -12 & 8 & 0 \\ -6 & -4 & 0 \\ 7 & -10 & 0 \end{bmatrix}$$

$\boxed{4}$ **a)** The system of equations can be written on the form $Ax = b$, with

$$A = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -8 \\ 3 \end{bmatrix}.$$

Gauss elimination yields

$$\begin{aligned} \begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ -1 & 1 & 2 & 3 \end{bmatrix} &\sim \begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 2 & 1 & -8 \\ 0 & -1 & -1 & 3 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & -3 \\ 0 & 2 & 1 & -8 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 1 & 3 \end{bmatrix}, \end{aligned}$$

and we obtain $x_3 = 2$, $x_2 = -5$ and $x_1 = -4$ by backward substitution. The solution $x = (-4, -5, 2)$ is unique.

b) We have

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 8 \end{bmatrix}.$$

By Gauss elimination we get

$$\begin{aligned} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix} &\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}. \end{aligned}$$

We can choose x_3 as we want, let's say $x_3 = t$, and then we get $x_2 = 3 - 2t$ and $x_1 = -2 + t$. The number of solutions are infinite, and they are on the form $x = (-2 + t, 3 - 2t, t) = (-2, 3, 0) + t(1, -2, 1)$ for $t \in \mathbb{R}$.

$\boxed{5}$ Since $|x|$ is non-negative, both $\mathbb{R}$ and $[0, \infty)$ are valid codomains. The set $(0, \infty)$ is *not* a valid codomain, because $f(0) = 0 \notin (0, \infty)$.

If f have $(0, \infty)$ as domain, all of the given sets are valid as codomains.