1. We find by Euler’s method

\[(x_1^{(1)}, x_2^{(1)}) = (\pi/4, 0) + 0.2(0, -\sin(\pi/4)) \approx (0.785, -0.141)\]
\[(x_1^{(2)}, x_2^{(2)}) \approx (0.785, -0.141) + 0.2(-0.141, -\sin(0.785)) \approx (0.757, -0.282)\]
\[(x_1^{(3)}, x_2^{(3)}) \approx (0.757, -0.282) + 0.2(-0.282, -\sin(0.757)) \approx (0.701, -0.419)\]

The angle decreases slightly (about 40° at \(t = 0.6\)), and the speed is negative (pendulum moving to the left). This is well aligned with what intuition tells us about pendulum movements.

2. We write the equation as

\[y^{-1/2}y' = t^2,\]

and we proceed to integrate it

\[2y^{1/2} = \frac{1}{3}t^3 + \tilde{C}.\]

Thus we get

\[y = (C + t^3/6)^2.\]

Plugging in the initial data, we get

\[y(t) = \left(1 + \frac{1}{6}t^3\right)^2.\]

3. Since \(\int 2/t \, dt = 2 \ln |t| + C = \ln(t^2) + C\), we can use \(\exp(\ln(t^2)) = t^2\) as an integrating factor. We find

\[(t^2y)' = \cos(t),\]

and therefore

\[t^2y(t) = \sin(t) + C.\]

Since \(y(\pi/2) = 0\) we have to choose \(C = -1\), and the solution of the initial value problem is therefore

\[y(t) = \frac{\sin(t) - 1}{t^2}.\]

(Note that this is a solution only for \(t > 0\).)
The characteristic polynomial is
\[ p(\lambda) = \lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2), \]
so that a basis for the homogeneous solutions is given by \( y_1(t) = e^t \) and \( y_2(t) = e^{2t} \).
Since the right hand side doesn’t solve the homogeneous equation, we try a particular solution on the form
\[ y_p(t) = Ae^{3t}. \]
Inserting this into the equation, we see that \( A = \frac{1}{2} \). The general solution is thus
\[ y(t) = \frac{1}{2} e^{3t} + \alpha e^t + \beta e^{2t}, \]
where \( \alpha, \beta \in \mathbb{R} \) are constants.

By careful calculations we get
\[
\begin{align*}
    y'_p(t) &= e^{-t} \cos(t) - te^{-t} \cos(t) - te^{-t} \sin(t) \\
    y''_p(t) &= -e^{-t} \cos(t) - e^{-t} \sin(t) - e^{-t} \cos(t) + te^{-t} \cos(t) + te^{-t} \sin(t) \\ & \quad - e^{-t} \sin(t) + te^{-t} \sin(t) - te^{-t} \cos(t) \\
    &= -2e^{-t} \cos(t) - 2e^{-t} \sin(t) + 2te^{-t} \sin(t),
\end{align*}
\]
and therefore
\[
\begin{align*}
    y'''_p(t) + 2y'_p(t) + 2y_p(t) &= -2e^{-t} \cos(t) - 2e^{-t} \sin(t) + 2te^{-t} \sin(t) \\ & \quad + 2e^{-t} \cos(t) - 2te^{-t} \cos(t) - 2te^{-t} \sin(t) \\ & \quad + 2te^{-t} \cos(t) \\
    &= -2e^{-t} \sin(t),
\end{align*}
\]
such that \( y_p \) is a solution of the equation.