Norwegian University of Science
and Technology
Department of Mathematical
Sciences

Introductory course in linear algebra and differential equations
Autumn 2019

Exercise set 1

1 Are the following functions linear transformations? If yes, write the corresponding matrix.

a) $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $S(x) = (x_1x_2, 2x_1)$

b) $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x) = (x_1 + 3x_2, 4x_3)$

c) $K\colon \mathbb{R}^3 \to \mathbb{R}^3$ defined by $K(x) = (x_2, x_1 + 2, x_3)$

d) $L\colon \mathbb{R}^2 \to \mathbb{R}^2$ defined by $L(x) = (-x_1, x_2)$

e) $P\colon \mathbb{R} \to \mathbb{R}$ defined by $P(x) = \sin(x)$

2 Describe what L from the previous exercise does to the $\mathbb{R}^2$-plane.

3 Calculate:

a)
$$2\begin{bmatrix}3 & 1 & 2\\ 6 & 1 & 2\\ 4 & 2 & 5\end{bmatrix} - \begin{bmatrix}6 & -3 & 2\\ 3 & 8 & 6\\ 6 & 5 & -2\end{bmatrix}$$

b)
$$\begin{bmatrix}-1 & 3\\ 4 & 0\end{bmatrix}\begin{bmatrix}1 & -3\\ 1 & -2\end{bmatrix}$$

c)
$$\begin{bmatrix}0 & 4\\ 3 & 4\\ 2 & -1\end{bmatrix}\begin{bmatrix}2 & -4 & 0\\ -3 & 2 & 0\end{bmatrix}$$

4 Write the set of equations on matrix form. Find the augmented matrix and solve by Gaussian elimination.

a)
$$\begin{aligned} x_1 - 2x_2 - 3x_3 &= 0\\ 2x_2 + x_3 &= -8\\ -x_1 + x_2 + 2x_3 &= 3\end{aligned}$$

b)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4\\ 5x_1 + 6x_2 + 7x_3 &= 8\end{aligned}$$

5 Which of the following sets can you take as codomain Y in $f\colon \mathbb{R} \to Y$ defined by $f(x) = |x|$ and get a well-defined function? What if the domain was $(0, \infty)$?

- $\mathbb{R}$
- $(0, \infty)$
- $[0, \infty)$