Introductory course in
linear algebra and
differential equations
Autumn 2019

Norwegian University of Science
and Technology
Department of Mathematical
Sciences

Exercise set 3

1 Calculate the determinant of the following matrices. Are they invertible?

a)
$$\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

b)
$$\begin{bmatrix} -2 & 2 & 2 \\ 3 & 3 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

c)
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 6 & 7 & 8 \\ 0 & 0 & 1 & 9 & 10 \\ 0 & 0 & 0 & 0 & 11 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

2 (From yesterday!) For which values of $a \in \mathbb{R}$ is the matrix

$$A = \begin{bmatrix} 1 & 0 & a \\ a & 1 & 0 \\ 0 & a & 1 \end{bmatrix}$$

invertible? Use the determinant.

3 Find the characteristic polynomial of the matrix

$$U = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}.$$

What are the eigenvalues?

(More generally, can you show that the eigenvalues of a $n \times n$ upper (or lower) triangular matrix are given by the numbers on the diagonal?)

4 Find the eigenvalues, their algebraic- and geometric multiplicities, and their corresponding eigenvectors for the following matrices:

a)
$$A = \begin{bmatrix} 5 & 2 \\ -3 & 0 \end{bmatrix}$$

b)
$$B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & -9 & 4 \end{bmatrix}$$