



- 1 In a metropolitan area with constant total population, 7 million people lives in the city centre and 5 million people lives in the suburbs. Every year 20 % of the inhabitants of the centre moves to the suburbs (80 % stays in the centre), while 10 % of the people living in the suburbs moves to the centre (90 % stays in the suburbs).

In the long term, what is the distribution of inhabitants between the centre and the suburbs?

**Hint:** The matrix

$$A = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{bmatrix}$$

can be diagonalized as

$$A = PDP^{-1},$$

with

$$P = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 0.7 \end{bmatrix}.$$

Furthermore, it holds that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

- 2 In this exercise we will study the symmetric matrix

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 4 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$

- Find the characteristic polynomial of  $A$  (on factorised form)  
(Hint: Choose wisely which column/row to start with.)
- Find all the eigenvalues of  $A$  and the corresponding eigenvectors.
- Find the orthogonal diagonalization of  $A$ .
- Find the matrix exponential  $e^A$  by using the orthogonal diagonalization of  $A$ .