Norwegian University of Science
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Department of Mathematical
Sciences

Introductory course in linear algebra and differential equations Autumn 2019

Exercise set 6

1 The movements of a pendulum can be described by the equation $\ddot{\theta} + \sin(\theta) = 0$. If we define $x_1 = \theta$ and $x_2 = \dot{\theta}$, we can write the equation as $\dot{x} = f(x)$ with $f(x_1, x_2) = (x_2, -\sin(x_1))$.

Assume that $x^{(0)} = (\pi/4, 0)$ (angle of 45°). Use Euler's method with step size $h = 0.2$ to find an approximation of $x^{(3)}$ of the solution at $t = 0.6$.

2 Use separation of variables to solve the initial value problem

$$y' - t^2\sqrt{y} = 0, \qquad y(0) = 1.$$

3 Solve the initial value problem

$$y' + \frac{2}{t}y = \frac{\cos(t)}{t^2}, \qquad y(\pi/2) = 0$$

by using integrating factor.

4 What is the general solution to the equation

$$y'' - 3y' + 2y = e^{3t}?$$

5 The equation

$$y'' + 2y' + 2y = -2e^{-t}\sin(t),$$

has a particular solution on the form

$$y_p(t) = Ate^{-t}\cos(t) + Bte^{-t}\sin(t).$$

Show that this is in fact a solution if $A = 1$ and $B = 0$ (plug the expression $y_p(t) = te^{-t}\cos(t)$ into the equation).