



Norwegian University of Science
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Department of Mathematical
Sciences

Introductory course in
linear algebra and
differential equations

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Exercise set 7

- 1 The salinity (amount of salt) x_1 and x_2 (in gram) in two connected tanks of water can be described by the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.01 & 0.01 \\ 0.01 & -0.01 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Find $x_1(t)$ og $x_2(t)$ if we start with 100 grams of salt in the first tank, and nothing in the other. At which time is there 25 grams of salt in tank number two?

- 2 The matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

has the eigenvalues 1 and 3 (one of these has multiplicity 2). Solve the following initial value problem:

$$\begin{aligned} \dot{x}_1 &= 2x_1 - x_2 + x_3 & x_1(0) &= 1 \\ \dot{x}_2 &= -x_1 + 2x_2 + x_3 & x_2(0) &= 2 \\ \dot{x}_3 &= 3x_3 & x_3(0) &= 1. \end{aligned}$$