Oppgaver

ØF II om

Projeksjoner

Oppg. 1 Finn ortogonal basis for

$U = Sp\{1, x, x^2\} \subset C[0,1]$ med indre produkt

$\langle f, g \rangle = \int_0^1 f(x) g(x)\, dx$

Oppg. 2. Finn $\text{proj}_U h(x)$, der $h(x) = e^x$.

$V_1 = 1$

$V_2 = x$

$V_3 = x^2$

$\xrightarrow{G-S}$

$U_1 = ?$

$U_2 = ?$

$U_3 = ?$

$U_1 = V_1 = 1$

$$U_2 = V_2 - \frac{\langle V_2, U_1 \rangle}{\langle U_1, U_1 \rangle} U_1 = x - \frac{1}{2}$$

$$\langle V_2, U_1 \rangle = \int_0^1 x\, dx = \frac{1}{2}$$

$$\langle U_1, U_1 \rangle = \int_0^1 1\, dx = 1$$

$$U_3 = V_3 - \frac{\langle V_3, U_1 \rangle}{\langle U_1, U_1 \rangle} U_1 - \frac{\langle V_3, U_2 \rangle}{\langle U_2, U_2 \rangle} U_2$$

$$\langle V_3, U_1 \rangle = \int_0^1 x^2 \, dx = \frac{1}{3}$$

$$\langle V_3, U_2 \rangle = \int_0^1 x^2 \left(x - \frac{1}{2}\right) dx = \int_0^1 x^3 - \frac{1}{2} x^2 \, dx$$

$$= \left[\frac{1}{4} x^4 - \frac{1}{6} x^3 \right]_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

$$\langle U_2, U_2 \rangle = \int_0^1 \left(x - \frac{1}{2}\right)^2 dx = \frac{1}{12}$$

$$U_3 = V_3 - \frac{\langle V_3, U_1 \rangle}{\langle U_1, U_1 \rangle} U_1 - \frac{\langle V_3, U_2 \rangle}{\langle U_2, U_2 \rangle} U_2$$

$$= x^2 - \frac{1}{3} - \left(x - \frac{1}{2}\right) = x^2 - x + \frac{1}{6}$$

$U_1 = 1$ $\quad$ $U_3 = x^2 - x + \frac{1}{6}$

$U_2 = x - \frac{1}{2}$

er en ortogonal basis

$U_1 = 1$ $\qquad$ $U_3 = x^2 - x + \frac{1}{6}$

$U_2 = x - \frac{1}{2}$

er en ortogonal basis

Finn $\quad proj_U h(x) \qquad h(x) = e^x$

$$proj_U h(x) = \frac{\langle h, U_1 \rangle}{\langle U_1, U_1 \rangle} U_1 + \frac{\langle h, U_2 \rangle}{\langle U_2, U_2 \rangle} U_2 + \frac{\langle h, U_3 \rangle}{\langle U_3, U_3 \rangle} U_3$$

$$\langle h, U_1 \rangle = \int_0^1 e^x \, dx = e - 1$$

$$\langle h, U_2 \rangle = \int_0^1 e^x \left(x - \frac{1}{2}\right) dx = \int_0^1 e^x x - \frac{1}{2} e^x \, dx$$

$$\boxed{\left(e^x(x-1)\right)' = e^x x} \qquad = \left[e^x(x-1) - \frac{1}{2} e^x \right]_0^1$$

$$= \frac{3 - e}{2}$$

$$\langle u, v_3 \rangle = \int_0^1 e^x \left(x^2 - x + \frac{1}{6}\right) dx$$

$$= \int_0^1 e^x x^2 - x e^x + \frac{1}{6} e^x \, dx$$

$$= \Big[e^x (x^2 - 2x + 2) - e^x (x - 1) + \frac{1}{6} e^x \Big]_0^1$$

$$= \frac{7e - 19}{6}$$

$$\boxed{\left(e^x (x^2 - 2x + 2)\right)' = e^x x^2}$$

$$\langle v_3, v_3 \rangle = \int_0^1 \left(x^2 - x + \frac{1}{6}\right)^2 dx = \frac{1}{180}$$

$$\operatorname{proj}_U h(x) = \frac{\langle h, U_1 \rangle}{\langle U_1, U_1 \rangle} U_1 + \frac{\langle h, U_2 \rangle}{\langle U_2, U_2 \rangle} U_2 + \frac{\langle h, U_3 \rangle}{\langle U_3, U_3 \rangle} U_3$$

$$= (e-1) + \frac{\frac{3-e}{2}}{\frac{1}{12}} \left(x - \frac{1}{2}\right) + \frac{\frac{7e-19}{6}}{\frac{1}{180}} \left(x^2 - x + \frac{1}{6}\right)$$

$$= e - 1 + (18 - 6e)x - (9 - 3e)$$

$$+ 30(7e - 19)\left(x^2 - x + \frac{1}{6}\right)$$

$$= (210e - 570)x^2 + (588 - 216e)x$$

$$+ (39e - 105)$$

$$\approx 0{,}84\,x^2 + 0{,}85\,x + 1$$