

Problems for First day

1. Verify that both

$$y_1(x) = 5e^x \text{ and } y_2(x) = 3e^x$$

solves

$$y'(x) = y(x)$$

Can you find more solutions? Sketch the solutions in the xy -plane.

2. Verify that both

$$y_1(t) = 3e^{-2t} \text{ and } y_2(t) = -2e^{3t}$$

are solutions of

$$y'' - y' - 6y = 0$$

Is $y(t) = 10y_1(t) + 2y_2(t)$ also a solution?

2. Determine for which real values $a \in \mathbb{R}$ the function $y(t) = \arctan(t)$ satisfy the differential equation

$$(t^2 + 1)y''(t) + aty(t) = 0$$

3. Find the equilibrium points and classify them as either stable or unstable for the ODE

$$y'(t) = f(t)$$

where

a) $f(x) = x \left(\frac{1}{2} - e^{-|x|} \right)$

b) $f(x) = |x| - 1$

c) $f(x) = \sin(x)$

4. Solve the initial value problem

$$y' + \frac{2}{t}y = \frac{\cos(t)}{t^2}, \quad y\left(\frac{\pi}{2}\right) = 0$$

5. Use separation of variables to solve the initial value problem

$$y' - t^2\sqrt{y} = 0, \quad y(0) = 1.$$

6. Solve the following ODEs

a) $y'(x) = \frac{3y-1}{x}$

b) $y'(x) + \frac{2y}{x} = \frac{1}{x^2}$

7. Find an equation for a curve going through the point $(2, 3)$ and which has slope $\frac{2x}{1+y^2}$.