



Contact during the exam:

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EXAM IN TMA4115 CALCULUS 3

English

Monday, August 9, 2004

Time: 0900–1400

Permitted aids (Code C): Approved calculator: (HP 30S),
Karl Rottmann: Matematisk Formelsamling.

Grades to be announced: September 1.

Justify all your answers.

Problem 1 Find all complex solutions of the equation

$$z^2 + (3 + 3i)z + 5i = 0.$$

Write the solutions on the form $z = x + iy$, and mark the solutions in the complex plane.

Problem 2 Find all real numbers x, y so that the complex number $z = x + iy$ satisfies

$$|z + i| = |z - 1|.$$

Sketch the solution set in the complex plane.

Problem 3 Explain why $y' = x^2 + y^2$ not is a linear first order differential equation. Use Euler's method with step length $h = 0.1$ to approximate $y(0.2)$ when

$$y' = x^2 + y^2, \quad y(0) = 1.$$

Problem 4 Find a linear homogen second order differential equation $y'' + ay' + by = 0$ with general solution

$$y = e^x (c_1 e^{x\sqrt{2}} + c_2 e^{-x\sqrt{2}}).$$

Problem 5 Solve the differential equations

a) $y'' + y = \cos x$

b) $y'' + y = 1/\cos x \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$

Problem 6 A tank contains to begin with 100 liter pure water. Then a brine containing 50 gram of salt per liter starts running into the tank at a rate of 2 liter per minute. The diluted brine leaves the tank at the same rate.

After 10 minutes the brine running into the tank is replaced by pure water. (The rate of the liquid in and out of the tank is as before 2 liter per minute.)

a) How much salt is there in the tank after 10 minutes?

b) How much salt is there in the tank after 20 minutes?

The salt mixture in the tank is kept uniform by stirring.

Problem 7

a) Solve the system of equations

$$\begin{aligned} x + 2y &= 1 \\ 3x + 7y + 2z &= 2 \\ 2x + 3y - 3z &= 0. \end{aligned}$$

b) Decide for which values of the parameters a and b the system of equations

$$\begin{aligned}x + 2y &= 1 \\3x + 7y + 2z &= b \\2x + 3y - az &= 0\end{aligned}$$

has

- i) exactly one solution
- ii) infinitely two solutions
- iii) infinitely many solutions
- iv) no solutions.

Problem 8 If the null space $\text{Null}(A)$ of a 5×6 -matrix A has dimension 4, what can you say about the dimension of the column space $\text{Col}(A)$ and the row space $\text{Row}(A)$?

Problem 9 Given vectors $\mathbf{v}_1 = [2, 1, 0, 0]$, $\mathbf{v}_2 = [3, 0, 1, 0]$, $\mathbf{v}_3 = [4, 1, 0, 1]$ and $\mathbf{b} = [0, 0, 0, 9]$. If $V = \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, what is then the orthogonal projection of \mathbf{b} onto

- a) V ?
- b) V^\perp ?

Problem 10

a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 7 & 1 \\ 1 & 7 \end{bmatrix}.$$

b) Make a shift of variables $\begin{bmatrix} x \\ y \end{bmatrix} = P \begin{bmatrix} x' \\ y' \end{bmatrix}$ that brings the quadratic form

$$7x^2 + 2xy + 7y^2$$

into a quadratic form without mixed terms $(x'y')$. Write down the matrix P and the new quadratic form.

Problem 11 Let A be a diagonalizable matrix where every eigenvalue is either 1 or -1 . Show that $A^{-1} = A$.