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EXAM IN TMA4110 CALCULUS 3

English
Wednesday December 20, 2006
Time: 9–13

You may use the following (code C): Approved calculator (HP30S)
Rottman: *Matematisk formelsamling*

Grades to be announced: January 19, 2007

All answers have to be justified (with the exception of Problem 3). When grading, the 12 problems (1, 2abc, 3, 4, 5abc, 6ab, 7) will, as a rule, have the same weight.

Problem 1 Solve the equation

$$z^3 = \frac{4}{1 - i\sqrt{3}}.$$

Write the solutions in the form $re^{i\theta}$. Use a figure to show the location of the solutions in the complex plane.

Problem 2 Find a general solution of the differential equations

a) $y'' - y' + 2.5y = 0,$

b) $y'' + y' - 2y = x + e^{-2x},$

c) $4x^2y'' + 8xy' - 3y = 0, \quad x > 0.$

Problem 3 *Multiple choice problem — to be answered without explanations by choosing one alternative for each question.*

Let $y_1(x)$ and $y_2(x)$ be two solutions of the constant coeffisient equation $y'' + ay' + by = 0$. If the Wronskian $W(y_1, y_2)$ equals 0 when $x = 0$, what is the value of $W(y_1, y_2)$ when $x = 1$?

A: 0 **B:** 1 **C:** e **D:** The value depends on a and b.

Which one of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$ is in the null space Null(A) of the matrix

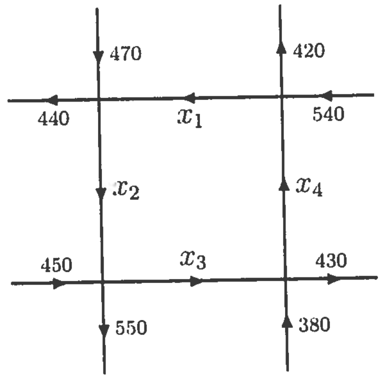
$$A = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & -3 \\ 2 & 1 & 1 \end{bmatrix}?$$

A: $\mathbf{v}_1 = (2, 1, 1)$ **B:** $\mathbf{v}_2 = (2, -3, 1)$ **C:** $\mathbf{v}_3 = (-2, 3, 1)$ **D:** $\mathbf{v}_4 = (-1, 2, 0)$

Problem 4 Four one-way streets in a city intesect as shown in the figure to the right. The number of cars passing per hour is shown in the figure.

Write down a system of equations in the form $A\mathbf{x} = \mathbf{b}$ which $\mathbf{x} = (x_1, x_2, x_3, x_4)$ must satisfy, and solve the system.

Find x_1, x_2 and x_3 if the x_4-section is closed for traffic, making $x_4 = 0$.



Problem 5 A matrix A and a vector $\mathbf{b}$ is given by

$$A = \begin{bmatrix} 1 & 1 & -2 & 0 & -1 \\ 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}.$$

a) Find a basis for the row space Row(A) and a basis for the column space Col(A).

b) What is the dimension of Row(A), Col(A) and the orthogonal complement Row$(A)^\perp$?

c) Show that the orthogonal complement Col$(A)^\perp$ is spanned by the vector $\mathbf{u} = (3, -1, 1, 3)$. Find, for example by using $\mathbf{u}$, a condition which b_1, b_2, b_3 and b_4 must satify in order that the system $A\mathbf{x} = \mathbf{b}$ shall be consistent.

Problem 6

a) Compute the eigenvalues of the matrix

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 0 \end{bmatrix}.$$

Find an orthogonal matrix P with determinant equal to 1 such that $P^T AP$ is a diagonal matrix.

b) A conic section has equation

$$3x^2 + 4xy - 1 = 0.$$

Introduce a new, rotated coordinate system such that the conic section is in standard position relative to the new coordinate system. Make a figure to show the location of the conic section and the new coordinate axes in the xy-plane.

Problem 7 Let A be an $n \times n$–matrix.

Show that if 5 is an eigenvalue of A, then 3 is an eigenvalue of $A - 2I$ where I denotes the $n \times n$ identity matrix.

Also show that if $\mathbf{v}$ is an n–vector satisfying $A\mathbf{v} \neq \mathbf{0}$ and $A^2\mathbf{v} = \mathbf{0}$, then $\mathbf{v}$ and $A\mathbf{v}$ are linear independant.