Norwegian University of Science and Technology
Department of Mathematical Sciences

Contact during the exam:
Ivar Amdal, phone 995 59 273
Cathrine Jensen, phone 454 42 047

EXAM IN TMA4115 CALCULUS 3

English
Wednesday, June 7, 2006
Time: 9–13

You may use the following (code C): Approved calculator (HP30S)
Rottman: *Matematisk formelsamling*

Grades to be announced: June 28.

All answers have to be justified (with the exception of Problem 4).

Problem 1 Calculate

$$w = \left(\sqrt{3} + i\right)^4.$$

Find the solution of the equation $z^4 = w$ lying in the *second* quadrant in the complex plane.

Problem 2

a) Solve the initial value problem

$$y'' + 6y' + 10y = 0, \quad y(0) = 0, \quad y'(0) = 2.$$

b) Find a general solution of the differential equation

$$y'' - 3y' + 2y = e^x + 10 \sin x.$$

c) Suppose that p and q are functions such that the differential equation

$$y'' + p(x)y' + q(x)y = 0, \qquad x > 0$$

has a basis $y_1 = x^2$, $y_2 = x^2 \ln x$ of solutions. Find a general solution of the equation

$$y'' + p(x)y' + q(x)y = x, \qquad x > 0.$$

Problem 3 Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ 2 & 4 & 3 & 3 & 1 \\ 1 & 2 & 2 & 1 & 1 \end{bmatrix}.$$

a) Solve the system of equations $A\mathbf{x} = \mathbf{0}$, and find a basis for $\mathrm{Null}(A)$.

b) Find a basis for each of the spaces $\mathrm{Col}(A)$, $\mathrm{Row}(A)$ and $\mathrm{Row}(A)^{\perp}$.

c) Show that the vector $\mathbf{v} = (2, 1, -1, 0)$ is in the orthogonal complement $\mathrm{Col}(A)^{\perp}$. Is $\{\mathbf{v}\}$ a basis for $\mathrm{Col}(A)^{\perp}$?

Problem 4 *Multiple choice problem — to be answered without explanations.*

a) Find the least squares solution $(\bar{x}, \bar{y})$ of the system

$$\begin{aligned} x + 3y &= 5 \\ x - y &= 1 \\ x + y &= 0. \end{aligned}$$

A: $(0, 1)$ **B:** $(1/2, 3/2)$ **C:** $(1, 1)$ **D:** $(3/2, 1/2)$

b) For which real numbers α, β is P an orthogonal matrix with determinant equal 1?

$$P = \frac{1}{2} \begin{bmatrix} \alpha & -1 & -1 & \beta \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ \alpha & 1 & 1 & \beta \end{bmatrix}$$

A: $\alpha = 2, \beta = 0$ **B:** $\alpha = 1, \beta = -1$ **C:** $\alpha = -1, \beta = 1$ **D:** $\alpha = 0, \beta = 2$

Problem 5 Given the matrix

$$A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -2 \\ 2 & -1 & -1 \end{bmatrix}.$$

a) Show that A has eigenvalues 0 and 1. Find all the eigenvectors of A.

b) Find an invertible matrix P and a diagonal matrix D such that

$$P^{-1}AP = D.$$

Solve the system of differential equations

$$\begin{aligned} y_1' &= 3y_1 - y_2 - 2y_3 \\ y_2' &= 2y_1 \phantom{{}-y_2} - 2y_3 \\ y_3' &= 2y_1 - y_2 - y_3 \end{aligned}$$

with initial conditions $y_1(0) = 0$, $y_2(0) = 1$ og $y_3(0) = 2$.

c) A square matrix B is said to be *idempotent* if $B^2 = B$. Show that the matrix A is idempotent. Show in general that if λ is an eigenvalue of an idempotent matrix, then $\lambda = 0$ or $\lambda = 1$.