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EXAM IN TMA4110 CALCULUS 3

English
Monday, December 3, 2007
9 am – 1 pm

You may use the following (code C): Approved calculator (HP30S)
Rottman: *Matematisk formelsamling*

Grades to be announced: January 3, 2008

All answers have to be justified. When grading, the 12 problems (1, 2abc, 3, 4, 5, 6abc, 7ab) will, as a rule, have the same weight.

Problem 1

Write down the polar form of the complex number $iz(\bar{z})^{-1}$, where $z = re^{i\theta}$ and $z \neq 0$.
Show on a figure all solutions of the equation $iz = \bar{z}$.

Problem 2

a) The equation

$$y'' + ay' + by = 0$$

with constant coefficients has solutions $y_1 = e^{(3/2)x}$ and $y_2 = e^{-(1/2)x}$. Determine a and b.

b) Solve the initial value problem

$$y'' - 4y' + 5y = 0, \qquad y(0) = 1,\ y'(0) = 4.$$

c) Find a general solution of the equation

$$y'' - 2y' - 15y = e^{5x} - 17\cos 3x.$$

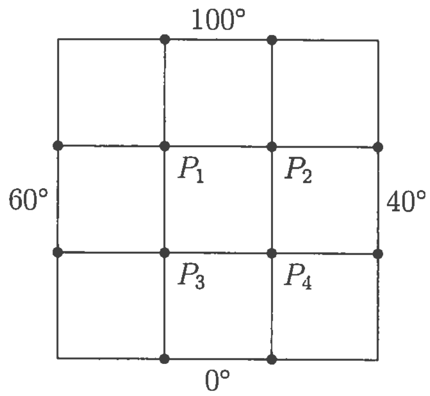
Problem 3 The equation

$$y'' + 2(\tan x)y' - y = 0$$

has a solution $y_1(x) = \sin x$. Find another solution y_2 of the form $y_2(x) = u(x)y_1(x)$ where u is a non-constant function.

The following integral is given: $\displaystyle\int \frac{\cos^2 x}{\sin^2 x}\,dx = -\frac{\cos x}{\sin x} - x + C.$

Problem 4 We shall determine the temperatures T_1, T_2, T_3, T_4 at four points P_1, P_2, P_3, P_4 on the square plate, see the figure on the right. Temperatures along the sides of the plate are shown on the figure.



Set up and solve a system of equations for T_1, T_2, T_3, T_4 if we assume that the temperature at each point P_1 to P_4 is the average of the temperatures at the four neighboring points (to the right, above, to the left, and below).

Problem 5 Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly independent vectors in $\mathbb{R}^n$. Are vectors

$$\mathbf{w}_1 = \mathbf{v}_1 + \mathbf{v}_2, \quad \mathbf{w}_2 = \mathbf{v}_1 + \mathbf{v}_3, \quad \mathbf{w}_3 = \mathbf{v}_2 + \mathbf{v}_3$$

linearly dependent or linearly independent? (Remember to explain the answer.)

Problem 6

a) Find a basis for the solution space of the homogeneous system

$$\begin{aligned} x_1 - 2x_2 + x_3 \phantom{{}+2x_4} &= 0 \\ x_1 \phantom{{}-2x_2} - x_3 + 2x_4 &= 0. \end{aligned}$$

b) Determine the orthogonal projection of the vector $(1, 2, -3, 1)$ onto the subspace $V = \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2\}$ of $\mathbb{R}^4$, where $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal vectors given by

$$\mathbf{v}_1 = (1, -2, 1, 0), \quad \mathbf{v}_2 = (1, 0, -1, 2).$$

c) Find also vectors $\mathbf{v}_3$ and $\mathbf{v}_4$ such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is an orthogonal basis for $\mathbb{R}^4$.

Problem 7 The matrix A is given by

$$A = \begin{bmatrix} 10 & -9 \\ 6 & -5 \end{bmatrix}.$$

a) Find the eigenvalues and eigenvectors of A, and write down a general solution of the system of differential equations

$$\begin{aligned} y_1' &= 10y_1 - 9y_2 \\ y_2' &= \ \ 6y_1 - 5y_2. \end{aligned}$$

b) Determine an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Find a matrix B such that $B^2 = A$.