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EXAM IN TMA4115 CALCULUS 3

English
Monday, May 21, 2007
9 am – 1 pm

You may use the following (code C): Simple calculator (HP30S) with user's manual
Rottman: *Matematisk formelsamling*

Grades to be announced: June 11, 2007

All answers have to be justified. When grading, the 12 problems (1, 2ab, 3ab, 4ab, 5ab, 6ab, 7) will, as a rule, have the same weight.

Problem 1

Show on a figure all complex numbers z such that

$$|z| = |z+1|.$$

Find the solutions of the equation

$$z^4 = (z+1)^4.$$

Problem 2

a) Solve the initial value problem

$$y'' + 2y' + y = 0, \quad y(0) = 2,\ y'(0) = 3.$$

b) Find a general solution of the differential equation

$$y'' + 2y' + y = x + 2e^{-x}.$$

Problem 3

a) Find a particular solution of the differential equation

$$x^2y'' - 4xy' + 4y = x^4, \quad x > 0.$$

b) Find a general solution of the differential equation

$$(y'' + 4y)'' = y'' + 4y.$$

Problem 4

The matrix $A = \begin{bmatrix} 1 & -1 & -2 & -2 & -4 \\ -2 & -4 & -2 & 1 & -1 \\ -2 & -1 & 1 & -2 & -1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} -2 \\ 1 \\ -2 \\ 0 \end{bmatrix}$ are given.

a) Solve the system $A\mathbf{x} = \mathbf{b}$.

b) Find a basis for $\text{Null}(A)$, $\text{Col}(A)$ and $\text{Row}(A)$.

Problem 5

Let the vectors $(1,0,1,-1)$, $(-1,1,0,1)$, $(1,1,0,1)$ be a basis for a subspace V of $\mathbb{R}^4$.

a) Use the Gram-Schmidt algorithm to find an orthogonal basis for V.

b) Find the orthogonal projection of $(1,1,3,4)$ into V.

Problem 6

a) Show that the matrix $A = \begin{bmatrix} -1 & -1 & 1 \\ -1 & -3 & 1 \\ -1 & -3 & 1 \end{bmatrix}$ has eigenvalues $0, -1, -2$.

Find a matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

b) Find the solution $x_1(t)$, $x_2(t)$, $x_3(t)$ of the system of differential equations

$$\begin{aligned} x_1' &= -x_1 - x_2 + x_3 \\ x_2' &= -x_1 - 3x_2 + x_3 \\ x_3' &= -x_1 - 3x_2 + x_3 \end{aligned}$$

for which $x_1(0) = 1$, $x_2(0) = -1$, $x_3(0) = 2$.

Problem 7

Let A and B be $m \times n$ matrices, and let C be a matrix such that $A = BC$. Show that $\text{Col}(A)$ is contained in $\text{Col}(B)$. Also show that if C is invertible, then the rank of A equals the rank of B.