NTNU
Norwegian University of
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Department of Mathematical Sciences

Examination paper for **TMA4110 Calculus 3**

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Examination date: December 7, 2019

Examination time (from–to): 09:00–13:00

Permitted examination support material: C: Specified printed and hand-written support material is allowed. A specific basic calculator is allowed.

Other information:
The exam consists of 10 subproblems. All subproblems are given equal weight. Give reasons for all answers. This year we specify that NO printed or handwritten support material is allowed.

Language: English

Number of pages: 3

Number of pages enclosed: 0

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Informasjon om trykking av eksamensoppgave	
Originalen er:	
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Date Signature

Problem 1

a) Solve the system of equations

$$\begin{aligned} x + y &= 2 \\ y + 2x &= 3 \\ -x + z &= 4 \end{aligned}$$

b) For each real number t, let

$$A_t = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ t & 0 & 1 \end{bmatrix}$$

For each value of t: Describe the column space and the null space of A_t.

Problem 2

Find the solution to the initial value problem

$$y'' - 3y' + 2y = e^{2t} \qquad y(0) = y'(0) = 1.$$

Problem 3

Let $A = \begin{bmatrix} r_1 & z \\ \bar{z} & r_2 \end{bmatrix}$ be a 2×2-matrix with $r_1, r_2 \in \mathbb{R}$ and $z \in \mathbb{C}$.

a) Show that the eigenvalues of A are real. Why is A diagonalizable for all values of r_1, r_2, z? What is such a matrix A called?

b) Let $r_1 = 3$, $r_2 = 2$ and $z = 1 + i$. Find the eigenvalues and eigenspaces of A. Find a matrix P, such that $P^{-1}AP$ is a diagonal matrix.

Problem 4

The students in Calculus 3 enjoy spending their weekends at Studentersamfundet, but sometimes they need a day at home to recharge their batteries. A quick show of hands in a lecture indicates that when going to Studentersamfundet, everyone prefers either fun and action at dance floor at the Bodega or cake and board games at Edgar. Attentive lecturers observe the following trend:

- If a student has been to the Bodega one weekend, there is a 50 % probability that he spends the following weekend home on his couch, and there is only a 10 % probability that you will meet him at the dance floor the following weekend.
- If a student spent the weekend at home, there is a one-fifth probability that the student is also at home the following weekend, and otherwise the probability that he goes to the Bodega equals the probability that he goes to Edgar.
- If a student went to Edgar one weekend, there is a 40 % probability that he will go to Edgar also the next weekend. Since the previous weekend was not too exhausting, there is a 35 % probability that the students chooses to show off at the dance floor. Some might feel more like having a quiet weekend at home - maybe turn on their TV and see if there is a late away-game with Rosenborg?

The first weekend of the semester, there is no entrance fee at Studentersamfundet and of course all students spend their Saturday there, evenly distributed between Edgar and the Bodega. How are the students distributed at the last weekend before the exam?

Problem 5

Let $V = P_2(\mathbb{R})$, that is: V is the vector space consisting of all polynomials of degree at most 2, with real coefficients. Let $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$ be defined by

$$\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)dx.$$

a) Give the definition of a real inner product space. Show that the function $\langle \cdot, \cdot \rangle$ gives an inner product of V.

b) We know that $\mathcal{B} = \{1, x, x^2\}$ is a basis for V. Use this basis to find an *orthogonal* basis $\mathcal{B}'$ for V. Let $r(x) = x^2 - 1$, and find $[r(x)]_{\mathcal{B}'}$, that is: the coordinates of $r(x) = x^2 - 1$ with respect to the basis $\mathcal{B}'$.

c) Let $T\colon V \to V$ be the function $T(p(x)) = p(x) + p'(x)$, where $p'(x)$ is the derivative of $p(x)$ with respect to x. Show that T is a linear transformation. Find a matrix A, with the property that $A[p(x)]_{\mathcal{B}} = [T(p(x))]_{\mathcal{B}}$.

d) Find a linear transformation which is the inverse of T, that is: a linear transformation $S\colon V \to V$ such that both compositions $S \circ T$ and $T \circ S$ equals the identity map on V.