Exam in TMA4110 Calculus 3

English
Thursday December 13, 2012
Time: 09:00 – 13:00
Grades ready by January 13, 2013

Permitted aids (Code C): Specified, simple calculator (HP 30S or Citizen SR-270X)
Rottmann: Matematisk formelsamling

All answers must be justified, and your calculations should be detailed enough to clearly indicate your line of argument. Each of the 8 problems has the same weight.

Problem 1 Show that $z_1 = 1 + \sqrt{3}i$ is a zero of the polynomial $P(z) = z^5 - 2z^4 + 4z^3 - 8z^2 + 16z - 32$ and find the 4 other zeros of $P$.

Problem 2 Find the general solution to the differential equation $y'' + 2y' + 5y = 2\cos t + 4\sin t$.

Problem 3 Find the general solution to the system

$3x_1 - 6x_2 + 6x_3 = -15$
$x_1 + x_2 + 4x_3 = 10$.

Problem 4 Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be an invertible linear transformation such that $T(x_1, x_2, x_3) = (x_2 + 2x_3, x_1 + 3x_3, 4x_1 - 3x_2 + 8x_3)$. Find a formula for $T^{-1}$. 
Problem 5  Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$. Find orthonormal bases for the column space $\text{Col}(A)$, the row space $\text{Row}(A)$, and the null space $\text{Nul}(A)$.

Problem 6  Let $P = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. Let $x_0, x_1, x_2, \ldots$ be the Markov chain defined by $x_0 = \begin{bmatrix} 0.4 \\ 0.6 \end{bmatrix}$ and $x_{i+1} = Px_i$ for $i = 0, 1, 2, \ldots$.

Find the steady-state vector for $P$ and an explicit formula for $x_i$.

Problem 7  Find the solution of the system

$$
\begin{align*}
x_1' &= x_1 + 3x_2 + 3x_3 \\
x_2' &= -3x_1 - 5x_2 - 3x_3 \\
x_3' &= 3x_1 + 3x_2 + x_3
\end{align*}
$$

that satisfies $x_1(0) = 1$, $x_2(0) = -1$ and $x_3(0) = 2$.

Problem 8  Find the equation $y = \beta_0 + \beta_1 x$ of the least-squares line that best fits the data points $(1,3)$, $(2,5)$, $(4,7)$ and $(5,9)$.