Norges teknisk–
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SIF5010 Matematikk 3 13.8.99

Fasit

Oppg 1

a) $y = c_1 + c_2 \cos x + c_3 \sin x - 2x \cos x$

b) $k = \pm 1; \qquad y = c_1 x + \dfrac{c_2}{x}, \quad x > 0$

c) $y = c_1 x + \dfrac{c_2}{x} + \left(\dfrac{2}{x} - 2\right) e^x, \qquad x > 0$

d) $m > 1$

Oppg 2

a) $A^{-1} = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

b) $\text{Rang}(M_a) = \begin{cases} 1 & \text{for } a = 1 \\ 2 & \text{for } a = -1 \\ 3 & \text{for } a \neq \pm 1 \end{cases}$

c) $\text{Dim Col}(M_a) = \begin{cases} 1 & \text{for } a = 1 \\ 2 & \text{for } a = -1 \\ 3 & \text{for } a \neq \pm 1 \end{cases} \qquad \text{Dim Null}(M_a) = \begin{cases} 3 & \text{for } a = 1 \\ 2 & \text{for } a = -1 \\ 1 & \text{for } a \neq \pm 1 \end{cases}$

$a = -1$:

Basis for $\text{Col}(M_a) : \{\mathbf{u_1}, \mathbf{u_2}\}$ der $\mathbf{u_1} = [1, 1, -1]^T, \mathbf{u_2} = [1, -1, 1]^T$ f.eks.

Basis for $\text{Null}(M_a) : \{\mathbf{v_1}, \mathbf{v_2}\}$ der $\mathbf{v_1} = [0, 1, 1, 0]^T, \mathbf{v_2} = [1, 0, 0, 1]^T$ f.eks.

Oppg 3

a) $\lambda_1 = 9$ med oppgitt egenvektor $\mathbf{k}_1 = [1, 1, 1]^T$;

$\lambda_{2,3} = 3$ med egenvektorer: $r\mathbf{k}_2 + s\mathbf{k}_3$ der $\mathbf{k}_2 = [-1, 1, 0]^T$,
$\mathbf{k}_3 = [-1, 0, 1]^T, (r, s) \neq (0, 0)$

b) $P = \dfrac{1}{\sqrt{6}} \begin{bmatrix} \sqrt{2} & 1 & \sqrt{3} \\ \sqrt{2} & 1 & -\sqrt{3} \\ \sqrt{2} & -2 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

c) $\mathbf{x} = c_1 \mathbf{k}_1 e^{9t} + c_2 \mathbf{k}_2 e^{3t} + c_3 \mathbf{k}_3 e^{3t}$

$\mathbf{x}(0) = [3, 0, 0]^T$ hvis $c_1 = 1$, $c_2 = c_3 = -1$

Oppg 4 **a)** Nei

Oppg 5 **a)** $P = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$; $\quad D = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$

b) Ellipse