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SIF5009 Matematikk 3 2.12.02

Fasit

Oppg 1 $z_k = 2e^{i(\frac{\pi}{6}+\frac{k\pi}{2})}, \quad k = 0, 1, 2, 3$

Oppg 2

a) $y = x^4 + x^{-4}, \quad x > 0$

b) $y = e^{-2x}\cos x + 2e^{-2x}\sin x$

c) $y = c_1e^x + c_2e^{2x} + 3 + 2x - xe^x$

d) $y = c_1e^{-3x} + c_2xe^{-3x} - \frac{1}{2}e^{-3x}\ln(1+x^2) + xe^{-3x}\arctan x$

Oppg 3

a) $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} + s\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad s, t \in \mathbb{R}$

b) $\text{Null(A)} = \text{span}\{(2,1,0,0,0),(1,0,1,1,0)\}$, (f.eks)
$\text{Col(A)} = \text{span}\{(1,2,1,2),(2,1,5,3),(2,3,4,-1)\}$, (f.eks)
$\text{Row(A)} = \text{span}\{(1,-2,0,1,0),(0,0,1,-1,0),(0,0,0,0,1)\}$, (f.eks)

c) $\mathbf{u_1} = \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u_2} = \begin{bmatrix} 1 \\ -2 \\ 6 \\ -5 \\ 0 \end{bmatrix}, \quad \mathbf{u_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ (f.eks)

Oppg 4

a) $\det A \neq 0 \quad \Leftrightarrow \quad a \neq 0, \pm 2$
$A^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

b) $\lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 3$
$P = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

c) $\mathbf{y} = c_1e^{-t}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + c_2e^{t}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_3e^{3t}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Oppg 6

$$y_1(t) = 5e^{-\frac{t}{100}} - 4e^{-\frac{t}{25}}$$
$$y_2(t) = 5e^{-\frac{t}{100}} + 2e^{-\frac{t}{25}}$$
$$T = \frac{100}{3}\ln 2$$