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SIF5010 Matematikk 3 30.05.02

Fasit

Oppg 1

(i) $z_k = 2^{\frac{1}{6}} e^{i(\frac{\pi}{12} + k\frac{2}{3}k\pi)}, \quad k = 0, 1, 2$

(ii) $z_1 = a, z_2 = 0$

Oppg 2

a) $y = (1 - \frac{1}{2}x)e^{-\frac{1}{2}x}$

b) $y = c_1 \cos x + c_2 \sin x - \frac{1}{2}\cos x$

c) $y = c_1 e^x + c_2 e^{2x} - e^x \ln(1 + e^x) + e^{2x}(x - \ln(1 + e^x))$

Oppg 3

a) $y = \frac{1}{x}(\ln(1 + x^n) - \ln 2), \quad x > 0$

b) $y(\frac{5}{2}) \approx \frac{177}{260} \approx 0.6808$

Oppg 4

a) $\mathbf{x} = t_1\mathbf{v_1} + t_2\mathbf{v_2} + t_3\mathbf{v_3} = t_1 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -9 \\ 0 \\ 7 \\ 1 \\ 0 \end{bmatrix} + t_3 \begin{bmatrix} 14 \\ 0 \\ -11 \\ 0 \\ 1 \end{bmatrix}$

$\text{Null(A)} = \text{span}(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_2})$

b) $\text{Col(A)} = \text{span}\{(1, 3, 4), -2, (1, 4, 5, -3)\}$

$\text{Row(A)} = \text{span}\{(1, 2, 1, 2, -3), (0, 0, 1, -7, 11)\}$

$\text{Row(A)}^\perp = \text{Null(A)} = \text{span}\{(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3})\}$

c) $\dim \text{Col(A)}^\perp = 2$

Oppg 6

a) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

b) $C = \begin{bmatrix} 6 & -3 & -3 \\ -3 & 2 & 1 \\ -3 & 1 & 2 \end{bmatrix}$

c) $\lambda_1 = 0, \quad \mathbf{v_1} = (1, 1, 1)$

$\lambda_2 = 1, \quad \mathbf{v_2} = (0, 1, -1)$

$\lambda_3 = 9, \quad \mathbf{v_3} = (-2, 1, 1)$

d) $y = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^t + c_3 \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} e^{9t}$

$\mathbf{a} = -(C - I)^{-1} A^{-1}(1, 0, 0)$