



SIF5010 Matematikk 3 30.05.03

Fasit

Oppg 1

$$\begin{aligned}z_0 &= e^{\frac{\pi}{3}i} = \frac{1}{2} + \frac{\sqrt{3}}{2}i \\z_1 &= e^{\frac{5\pi}{6}i} = -\frac{\sqrt{3}}{2} + \frac{\sqrt{1}}{2}i \\z_2 &= e^{\frac{4\pi}{3}i} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \\z_3 &= e^{\frac{11\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{\sqrt{1}}{2}i\end{aligned}$$

Oppg 2

Kurven er parabelen $y^2 = 4x$.

Oppg 3

a) $y(0.2) \approx 0.202$
b) $y = e^{x^2} \int_0^x e^{-t^2} dt$
 $\int_0^{0.2} e^{-t^2} dt \approx 0.1941$

Oppg 4

a) $y = 3e^{-x} + e^{2x} + xe^{2x}$
b) $y = c_1 e^x + c_2 x e^x + x e^x \ln x$
c) $\omega = 3$

Oppg 5

- a) 1) Nøyaktig en løsning: $\alpha \neq 1$.
2) Uendelig mange løsninger: $\alpha = 1$ og $\beta = 2$.
3) Ingen løsninger: $\alpha = 1$ og $\beta \neq 2$.

$$\text{Med } \alpha = 1 \text{ og } \beta = 2: \mathbf{x} = \begin{bmatrix} 4 \\ -1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}$$

b) $\text{Null}(A) : \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$
 $\text{Col}(A) : \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}$

$$\text{Row}(A) : \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Oppg 6 a) $\lambda_1 = \lambda_2 = -1 : \mathbf{x} = s \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad (s, t) \neq (0, 0).$

$$\lambda_3 = 5 : \mathbf{x} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad t \neq 0.$$

b) $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix} \text{ og } D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

c) $y = c_1 e^{-t} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_3 e^{5t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Oppg 8 a) $x_n = 3 + 2 \left(\frac{2}{5}\right)^n$
 $y_n = 6 - 2 \left(\frac{2}{5}\right)^n$

$$n \rightarrow \infty : \begin{bmatrix} x_n \\ y_n \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

b) $B = \frac{1}{5} \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$

$$\mathbf{y} = 2e^{-\frac{3}{5}t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$t \rightarrow \infty : \mathbf{y} \rightarrow \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$