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SIF5010 Matematikk 3 30.05.03

Fasit

Oppg 1

$z_0 = e^{\frac{\pi}{3}i} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
$z_1 = e^{\frac{5\pi}{6}i} = -\frac{\sqrt{3}}{2} + \frac{\sqrt{1}}{2}i$
$z_2 = e^{\frac{4\pi}{3}i} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
$z_3 = e^{\frac{11\pi}{6}i} = \frac{\sqrt{3}}{2} - \frac{\sqrt{1}}{2}i$

Oppg 2 Kurven er parabelen $y^2 = 4x$.

Oppg 3

a) $y(0.2) \approx 0.202$
b) $y = e^{x^2} \int_0^x e^{-t^2} dt$
$\int_0^{0.2} e^{-t^2} dt \approx 0.1941$

Oppg 4

a) $y = 3e^{-x} + e^{2x} + xe^{2x}$
b) $y = c_1 e^x + c_2 x e^x + x e^x \ln x$
c) $\omega = 3$

Oppg 5

a) 1) Nøyaktig èn løsning: $\alpha \neq 1$.
2) Uendelig mange løsninger: $\alpha = 1$ og $\beta = 2$.
3) Ingen løsninger: $\alpha = 1$ og $\beta \neq 2$.

Med $\alpha = 1$ og $\beta = 2$: $\mathbf{x} = \begin{bmatrix} 4 \\ -1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}$

b) Null(A) : $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

Col(A) : $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}$

$$\text{Row}(A): \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Oppg 6 **a)** $\lambda_1 = \lambda_2 = -1 : \mathbf{x} = s \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad (s,t) \neq (0,0).$

$\lambda_3 = 5 : \mathbf{x} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad t \neq 0.$

b) $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$ og $D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

c) $y = c_1 e^{-t} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_3 e^{5t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Oppg 8 **a)** $x_n = 3 + 2\left(\frac{2}{5}\right)^n$
$y_n = 6 - 2\left(\frac{2}{5}\right)^n$

$n \to \infty : \begin{bmatrix} x_n \\ y_n \end{bmatrix} \to \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

b) $B = \frac{1}{5} \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$

$\mathbf{y} = 2e^{-\frac{3}{5}t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$t \to \infty : \mathbf{y} \to \begin{bmatrix} 3 \\ 6 \end{bmatrix}$