



SIF5010 Matematikk 3 22.5.99

Fasit

- Oppg 1**
- a) $z_0 = 1 + i\sqrt{3}, z_1 = -2, z_2 = 1 - i\sqrt{3}$
 - b) $y = c_1 e^x + c_2 e^{-2x} + c_3 e^x \cos \sqrt{3}x + c_4 e^x \sin \sqrt{3}x + \frac{1}{9} x e^x$
 - c) $y = c_1 \cos 2x + c_2 \sin 2x + \cos 2x \ln(\cos 2x) + 2x \sin 2x$

Oppg 2 $y(\frac{5}{2}) \approx 4$

Oppg 3 a) $A \sim I$ hvis $\alpha \neq 0$, $A \sim \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ hvis $\alpha = 0$

- b) Nøyaktig en løsning $\Leftrightarrow \alpha \neq 0$
ingen løsning $\Leftrightarrow \alpha = 0$ og $\beta \neq 3$
uendelig mange løsninger $\Leftrightarrow \alpha = 0$ og $\beta = 3$

c) $\mathbf{x} = t \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 5 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad t \in \mathbf{R}$

d) Basis for $\text{Null}(A)$: $[-2, 1, 1, 0]^T$ (f.eks.)

Basis for $\text{Col}(A)$: $[1, 1, 1, 1]^T, [0, 1, 1, 1]^T, [4, 4, 0, 5]^T$ (f.eks.)

Basis for $\text{Null}(A)^\perp$: $[1, 0, 2, 0]^T, [0, 1, -1, 0]^T, [0, 0, 0, 1]^T$ (f.eks.)

Oppg 4 a) Egenverdier: 0, 2, 5

Egenvektorer: $t \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (t \neq 0)$

b) $P = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad (\text{f.eks.})$

Oppg 5 a) $\mathbf{x} = 20e^{-3t/100} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + 20e^{-t/50} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

b) $\mathbf{x} = 10e^{-t/25} \left(\frac{t}{100} \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right) + 10e^{-t/50} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$