Norwegian University of Science and Technology
Department of Mathematical Sciences

EXAM IN TMA4110/4115 MATHEMATICS 3
English
Thursday 9th August 2012
Time: 9-13

Examination Aids (code C): Simple calculator (HP30S or Citizen SR-270X)
Rottman: *Matematisk formelsamling*

Sensur: 30th August 2012

Give reasons for all answers, ensuring that it is clear how the answer has been reached. Each of the 7 problems has the same weight.

Problem 1 Write all of the solutions of $z^3 = 1$ in the form $z = x + iy$.

Write the solutions of $z^3 = \frac{-3+i}{\sqrt{2}(2+i)}$ in the form $z = x+iy$ and draw the solutions in the complex plane.

Problem 2 Find the solution of $y'' - 2y' + y = \frac{e^x}{x}$ for $x > 0$ which satisfies $y(1) = y'(1) = 0$.

Problem 3 The movement of a mechanical system satisfies the differential equation $y'' + 2cy' + 4y = 0$ for some constant $c > 0$.
For which values of c is the motion underdamped, overdamped, or critically damped?

Find the steady-state solution y_s of the equation $y'' + 2y' + 4y = \cos t$.

Problem 4 Let $T : \mathbb{R}^4 \to \mathbb{R}$ be the linear transformation given by

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = 2x + 2y - z + w.$$

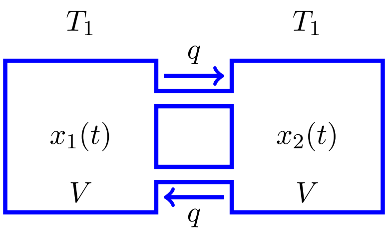
Find an orthonormal basis for the null space of T.

Problem 5 Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -t & 1 & 0 & 1 \\ 0 & -t & 1 & 0 \\ 0 & 0 & -t & -1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$.

For which values of t does the equation $A\mathbf{x} = \mathbf{b}$ have none, one, or infinitely many solutions?

Problem 6 For which numbers a does $\mathbb{R}^2$ have a basis of eigenvectors of the matrix $\begin{bmatrix} 0 & a \\ 1 & 0 \end{bmatrix}$?

Problem 7 Two water tanks, T_1 and T_2, each with volume $V = 100$ litre, are connected together with pipes as shown in the figure below.



The tanks are filled with salt water; $x_1(t)$ and $x_2(t)$ are the mass in grammes of salt in the respective tanks at time t. Salt water flows from tank T_1 to tank T_2, and equally from T_2 to T_1, at the rate $q = 1$ litres per second in each direction. We ignore the volume of the pipes, and assume instantaneous mixing of salt water (that is to say that the concentration of salt in each tank is constant throughout the tank at each moment in time).

Show that $x_1(t)$ and $x_2(t)$ satisfy the system:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}' = \frac{1}{100} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

and find $x_1(t)$ and $x_2(t)$ when $x_1(0) = 100$ g and $x_2(0) = 0$ g.

At what time is $x_2(t) = 25$ g?