Norwegian University of Science and Technology
Department of Mathematical Sciences

Contact during the examination:
Magnus Landstad tlf. 47259811
Eugenia Malinnikova tlf. 47055678

EXAMINATION TMA4110 CALCULUS 3
English
Wednesday, 1 December 2010
Time: 9-13

Permitted aids (code C): Simple calculator (HP30S or Citizen SR-270X)
Rottman: *Matematisk formelsamling*

Results: 22 December 2010

All answers should be justified: it should be made clear how the answer was obtained. Each of the 12 problem parts (1, 2a, 2b, 3a, 3b, 4a, 4b, 4c, 5, 6a, 6b, 7) counts equally.

Problem 1 Write down the polar form of the complex number $w = \frac{3-i}{2i-1}$. Find all of the solutions of the equation $z^4 = w$ and draw the solutions on the complex plane.

Problem 2

a) The motion of a mechanical system is described by the differential equation

$$y'' + 6y' + 18y = 0.$$

Determine whether the motion is under-damped, is over-damped or is critically damped. Find a particular solution $y(t)$ that satisfies the initial conditions $y(0) = 0$, $y'(0) = 0.6$.

b) Find the steady-state solution of the equation

$$y'' + 6y' + 18y = 45\cos 3t.$$

Problem 3

a) Find the general solution of the equation

$$y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 0, \ x > 0.$$

b) Find a particular solution of the equation

$$y'' - \frac{4}{x}y' + \frac{6}{x^2}y = x^2e^x, \ x > 0.$$

Problem 4 Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}.$$

Show that A is invertible and find A^{-1}.

Problem 5 Let $V \subset \mathbf{R}^4$ be the solution space of the system of equations

$$\begin{aligned} x + y - z + w &= 0 \\ x + 2y - 2z + w &= 0 \end{aligned}$$

a) Find an orthogonal basis for V.

b) Find the orthogonal projection of $b = (1, 1, 1, 1)$ onto V.

c) Find an orthogonal basis for $\mathbf{R}^4$ where the first two vectors of this basis are the vectors you found in part a).

Problem 6 Let

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -t & 1 & 0 & 1 & 1 \\ 0 & -t & 1 & 0 & 1 \\ 0 & 0 & -t & -1 & 0 \end{bmatrix}.$$

a) Find the rank of M for each value of t.

b) For which values of t is there a 5×4 matrix L such that $ML = I$, where I is the identity matrix? (Remember to justify your answer).

Problem 7 The equation

$$3x^2 - 2xy + 3y^2 = 1$$

describes a conic section in xy-plane. Find a rotated coordinate system (x', y'), in which the equation of the conic section is of the form

$$\lambda_1(x')^2 + \lambda_2(y')^2 = 1.$$

What kind of conic section is it? Draw the new coordinate axis and the conic section in the xy-plane.