NTNU
Norwegian University of Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4110 Matematikk 3**

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Examination date: 2nd December 2017

Examination time (from–to): 09:00–13:00

Permitted examination support material: C: Simple Calculator (Casio fx-82ES PLUS, Casio fx-82EX , Citizen SR- 270X, Citizen SR-270X College, or Hewlett Packard HP30S), Rottmann: Matematisk formelsamling.

Other information:
Give reasons for all answers, ensuring that it is clear how the answers have been reached.

Language: English

Number of pages: 3

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave	
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Date Signature

Problem 1

a) Write the complex number $z = -1 + i\sqrt{3}$ in polar form.

b) Show that $z = -1 + i\sqrt{3}$ is a sixth root of 64.

c) Sketch all solutions of $z^6 = 64$ on the complex plane.

Problem 2 Let $M = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 2 & 3 & -1 \\ -3 & -6 & -9 & 6 \\ 2 & 5 & 8 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 5 \end{bmatrix}$.

a) Show that $M\mathbf{x} = \mathbf{b}$ has infinitely many solutions.

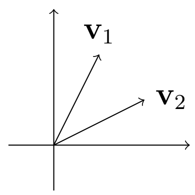
b) Write $\mathbf{b}$ as a linear combination of the columns of M.

c) Compute a basis for each of the following: $\operatorname{Col} M$, $\operatorname{Row} M$, and $\operatorname{Nul} M$.

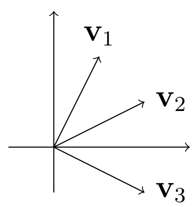
d) Find the determinant of M.
Hint: There is a shortcut. Therefore, expansion by cofactors is not necessary.

Problem 3 The two pictures show vectors in $\mathbb{R}^2$.

a) Are the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ linearly independent? Do they span $\mathbb{R}^2$? Are they a basis of $\mathbb{R}^2$? Justify your answers.



b) Are the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ linearly independent? Do they span $\mathbb{R}^2$? Are they a basis of $\mathbb{R}^2$? Justify your answers.



Problem 4 Consider the equation for an undamped forced harmonic motion:

$$y''(t) + y(t) = \cos(t - 2).$$

a) Find the general solution for the homogeneous equation.

b) Find the general solution of the inhomogeneous equation.

Problem 5 Let $A = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

a) Find the solution of the initial value problem

$$\mathbf{y}'(t) = A\mathbf{y}(t), \qquad \mathbf{y}(0) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

b) Show that the solution to the initial value problem

$$\mathbf{y}'(t) = A\mathbf{y}(t), \qquad \mathbf{y}(0) = \mathbf{y}_0$$

is unique.
Hint: Uniqueness of solutions means, that for each given initial value $\mathbf{y}_0$ there exists **exactly one** solution $\mathbf{y}$.

c) Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Explicitly state P^{-1}.
Hint: A is symmetric.

Problem 6 Let $W = \text{span}\{\mathbf{u}, \mathbf{v}\}$, where

$$\mathbf{u} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{v} = \begin{bmatrix} 4 \\ -4 \\ 2 \end{bmatrix}.$$

Find an orthonormal basis for W.

Problem 7 Let $\mathbf{u} = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$. Denote by $T : \mathbb{R}^3 \to \mathbb{R}^3$ the mapping

$$T(\mathbf{x}) = \frac{\mathbf{x} \cdot \mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u},$$

which is the orthogonal projection onto the subspace spanned by $\mathbf{u}$.

a) State the definition of a linear transformation.

b) Show that T is a linear transformation.
Hint: You can use without proof that the dot product is linear.

c) Find the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^3$.