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English version

TMA4115 Matematikk 3

Tid:
Examination Aids: D
No written and handwritten examination support materials are permitted.
Calculator: Citizen SR-270X or Hewlett Packard HP30S

Problem 1.

1. Show that $|\operatorname{Re} z| \leq |z|$.
2. Solve the equation $z^2 - 2\,\mathrm{i}\,z - 1 - 2\,\mathrm{i} = 0$. Write your answer in the form $x + \mathrm{i}\,y$.

Problem 2.

1. Solve the initial-value problem
$$y'' - 4y' + 3y = 0, \qquad y(0) = 1,\ y'(0) = 2.$$
2. Find a general solution of the differential equation
$$y'' - 4y' + 3y = 3 - 4e^x.$$

Problem 3.

$y = x$ is a solution of $y'' - (3x^2 + 4x^{-1})y' + (3x + 4x^{-2})y = 0$, find another solution (such that the two are linealry independent).

Problem 4.

An underdamped spring (with mass 1) has equation of motion:

$$y'' + cy' + ky = 0$$

Two solutions of this differential equation are

$$y_1 = e^{\lambda t}\cos(\omega t), \qquad y_2 = e^{\lambda t}\sin(\omega t)$$

1. Compute the Wronskian $W(y_1, y_2)$ and find a formula which uses c and k instead of λ and ω.
2. Asume that the time between successive maxima is $2s$, and that the maxsimum amplitude is reduced to $1/4$ of its first value after 15 oscillations. Find the damping constant of the system.

Problem 5.

Multiple-choice question, answer without showing your reasoning with one alternative for each question.

Let A be a 4×3-matrix. What is Rank A? (Which alternative is always right?)

A: at most 3 B: 3 C: at least 3 D: 4

Which alternative is the least-squares solution $(\overline{x}, \overline{y})$ of the linear system

$$-x + y = 5, \ -x + 2y = 0, \ -3x + y = -5 \ ?$$

A: $(2, 3/2)$ B: $(1, 1)$ C: $(3/2, 3/2)$ D: $(2, 2)$

Problem 6.

Let A be the following matrix; find a basis for each of the spaces Null(A), Col(A), Col$(A)^{\perp}$, and Row(A).

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 2 & 1 \\ 3 & 6 & 1 & 0 & 2 & -1 \\ 4 & 8 & 2 & -2 & 0 & -4 \end{bmatrix}$$

Find the orthogonal projection of $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ on to Col(A).

Problem 7.

Let

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

1. Find the eigenvalues and eigenvectors of A.
2. Find a matrix P and a diagonal matrix D such that $A = PDP^T$.
3. Solve the system of differential equations

$$\begin{aligned} y_1' &= 3y_1' + y_2' + y_3' \\ y_2' &= y_1' + 2y_2' \\ y_3' &= y_1' + 2y_3' \end{aligned}$$

with initial position $y_1(0) = 3$, $y_2(0) = 2$, $y_3(0) = -2$.

Problem 8.

1. Let
$$A = \begin{bmatrix} 0 & k \\ 0 & 0 \end{bmatrix}$$
Show that $A^2 = 0$ and that $I + A$ is invertible.
2. Let B be an $n \times n$-matrix such that $B^2 = 0$. Show that $I + B$ is invertible. Is B diagonalisable?