NTNU – Trondheim
Norwegian University of
Science and Technology

Department of Mathematical Sciences

Examination paper for TMA4115 Matematikk 3

Academic contact during examination: Andrew Stacey[a], Dag Wessel-Berg[b], Alexander Schmeding[c], Antoine Julien[d]

Phone: [a] 73590154 , [b] 73591343 , [c] 40539912 , [d] 73591773

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Examination time (from–to): 09:00–13:00

Permitted examination support material: C: Simple Calculator (Casio fx-82ES PLUS, Citizen SR-270X, Citizen SR-270X College, or Hewlett Packard HP30S), Rottmann: Matematiske formelsamling

Other information:
Give reasons for all answers, ensuring that it is clear how the answer has been reached. Each of the parts has the same weight.

Language: English

Number of pages: 2

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Checked by:

Date Signature

Problem 1 Find all solutions of $\operatorname{Im} z = \frac{\sqrt{2}}{2}|z|$ and draw them on the complex plane.

Problem 2 Find a particular solution of the differential equation:

$$y''(t) + 3y'(t) + 2y(t) = 2e^{-t}.$$

Problem 3 Given that $y_1(x) = x^{-1}$ and $y_2(x) = x^{-2}$ are two linearly independent solutions of the differential equation

$$y''(x) + 4x^{-1}y'(x) + 2x^{-2}y(x) = 0, \qquad x > 0,$$

and find the general solution of the differential equation:

$$y''(x) + 4x^{-1}y'(x) + 2x^{-2}y(x) = x^{-3}, \qquad x > 0.$$

Problem 4 Let

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 3 \\ -2 & -6 & 5 & 1 & -8 \\ 2 & 6 & 2 & 6 & -2 \\ -1 & -3 & 0 & -2 & 2 \end{bmatrix}$$

Find bases for $\operatorname{Null}(A)$, $\operatorname{Col}(A)$, and $\operatorname{Row}(A)$. What is $\dim(\operatorname{Null}(A^T))$?

Problem 5 For which values of a is the following family of vectors linearly independent? For each a where the family is linearly dependent, give a non-trivial linear dependency between the vectors.

$$\left\{ \begin{bmatrix} a \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ a \\ 3 \end{bmatrix}, \begin{bmatrix} a \\ -1 \\ 0 \end{bmatrix} \right\}$$

Problem 6 Let

$$A = \begin{bmatrix} 3 & 7 & 5 \\ 4 & 1 & -10 \\ 0 & 1 & 2 \end{bmatrix}$$

Find an orthogonal basis for $\mathbb{R}^3$ that begins with an orthogonal basis for $\operatorname{Col}(A)$.

Problem 7 The newly opened student cafeteria at the Antarctic University of Tropical Medicine offers a choice of three meals: a meat dish, a vegetarian dish, and a sandwich meal. Extensive research reveals that students select their meal based solely on what they ate the day before. There is a probability of 0.1 that a student will eat the same as the previous day. If a student did not have the meat dish yesterday, then the probability that they will choose the meat dish today is 0.3. If a student did have the meat dish yesterday, then they will choose the sandwich today with probability 0.3.

Set up the stochastic matrix for this situation and use it to find the proportions that the caterers should buy the meals in the long term.

Problem 8 Find the eigenvalues and eigenvectors (which might be complex) of the matrix

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & -2 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

Find the solution of the following system of differential equations with initial conditions $x_1 = 0, x_2 = 0, x_3 = 1$:

$$\begin{aligned} x_1' &= -x_3 \\ x_2' &= x_1 - 2x_2 + 2x_3 \\ x_3' &= x_1 \end{aligned}$$

Write your answer in terms of real functions.

Problem 9 A scientist records the following data for her experiment:

Control (x)	-2	-1	0	1	2
Reading (y)	5	4	2	4	15

The model for this data is a quadratic $y = ax^2 + bx + c$. Find the least-squares solution for a, b, c that best fits the data.

Problem 10 Let A be an $m \times n$–matrix. Explain why $A^T A$ is a symmetric matrix. What size is it?

Use the spectral theorem to show that there is an orthogonal basis $\{\vec{v}_j\}$ for $\mathbb{R}^n$ such that $\{A\vec{v}_j\}$ is an orthogonal family (i.e. the vectors are pairwise orthogonal).