Department of Mathematical Sciences

Examination paper for **TMA4115 Matematikk 3**

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Examination date: 26th May, 2015
Examination time (from–to): 09:00-13:00
Permitted examination support material: C: Simple calculator (Casio fx-82ES PLUS, Citizen SR-270X or Citizen SR-270X College, Hewlett Packard HP30S), Rottmann: *Matematisk formelsamling*

Other information:
Give reasons for all answers, ensuring that it is clear how the answer has been reached. Each of the 12 problem parts has the same weight when grading.

Language: English
Number of pages: 2
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Checked by:

Date Signature

Problem 1 Solve the quadratic equation $z^2 + (4 + 2i)z + 3 = 0$, write the solutions in normal form.

Problem 2

a) Solve the initial value problem

$$x'' + 6x' + 8x = 0, \quad x(0) = 0,\ x'(0) = 8.$$

What is the maximal value attained by this solution $x(t)$ for $t > 0$?

b) Find the steady-state solution of the equation

$$x'' + 6x' + 8x = 4\cos 2t.$$

Problem 3 Find general solution of the equation

$$y'' + y = 3x + \tan(x).$$

(Hint $\int(\cos x)^{-1}dx = \ln|\sec x + \tan x|$.)

Problem 4 Let

$$A = \begin{bmatrix} 1 & t \\ t & 2 \end{bmatrix}.$$

a) For which values of t does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for any $\mathbf{b}$ in $\mathbb{R}^2$?

b) Find an LU decomposition of A (the result will depend on the parameter t).

Problem 5 Given the following vectors in $\mathbb{R}^4$

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 4 \\ -3 \\ -2 \\ 4 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 3 \\ -2 \\ 1 \\ 1 \end{pmatrix},$$

let $V = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

a) Are the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ linearly independent? Find a basis for V.

b) Find an orthogonal basis for V.

c) Does there exist a vector $\mathbf{u} \neq \mathbf{0}$ in $\mathbb{R}^4$ which is orthogonal to $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$?

Problem 6

a) Find (complex) eigenvalues and (complex) eigenvectors of the matrix

$$\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

b) Find the solution of the system

$$\begin{aligned} x_1' &= x_1 - 2x_2 \\ x_2' &= x_1 + 3x_2 \end{aligned}$$

that satisfies the initial conditions $x_1(0) = 1$ and $x_2(0) = 1$. Write down the answer using real-valued functions.

Problem 7 Suppose that A is an $m \times n$-matrix with real entries. Prove that $\mathbf{x} \cdot A^T A\mathbf{x} \geq 0$ for each $\mathbf{x}$ in $\mathbb{R}^n$ and therefore each real eigenvalue of the matrix $A^T A$ is non-negative.