Examination paper for **TMA4115 Matematikk 3**

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**Examination date:** 26th May, 2015
**Examination time (from–to):** 09:00-13:00

**Permitted examination support material:** C: Simple calculator (Casio fx-82ES PLUS, Citizen SR-270X or Citizen SR-270X College, Hewlett Packard HP30S), Rottmann: *Matematisk formelsamling*

**Other information:**
Give reasons for all answers, ensuring that it is clear how the answer has been reached. Each of the 12 problem parts has the same weight when grading.

**Language:** English
**Number of pages:** 2
**Number pages enclosed:** 0

Checked by:

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Date Signature
Problem 1  Solve the quadratic equation $z^2 + (4 + 2i)z + 3 = 0$, write the solutions in normal form.

Problem 2

a) Solve the initial value problem

$$x'' + 6x' + 8x = 0, \quad x(0) = 0, \quad x'(0) = 8.$$ What is the maximal value attained by this solution $x(t)$ for $t > 0$?

b) Find the steady-state solution of the equation

$$x'' + 6x' + 8x = 4 \cos 2t.$$  

Problem 3  Find general solution of the equation

$$y'' + y = 3x + \tan(x).$$  

(Hint $f(\cos x)^{-1}dx = \ln |\sec x + \tan x|$.)

Problem 4  Let

$$A = \begin{bmatrix} 1 & t \\ t & 2 \end{bmatrix}.$$ 

a) For which values of $t$ does the equation $Ax = b$ have a solution for any $b$ in $\mathbb{R}^2$?

b) Find an LU decomposition of $A$ (the result will depend on the parameter $t$).

Problem 5  Given the following vectors in $\mathbb{R}^4$

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 4 \\ -3 \\ -2 \\ 4 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 3 \\ -2 \\ 1 \\ 1 \end{pmatrix},$$

let $V = \text{Span}\{v_1, v_2, v_3, v_4\}$.

a) Are the vectors $\{v_1, v_2, v_3, v_4\}$ linearly independent? Find a basis for $V$.

b) Find an orthogonal basis for $V$.

c) Does there exist a vector $u \neq 0$ in $\mathbb{R}^4$ which is orthogonal to $v_1, v_2, v_3, v_4$?
Problem 6

a) Find (complex) eigenvalues and (complex) eigenvectors of the matrix

\[
\begin{bmatrix}
1 & -2 \\
1 & 3
\end{bmatrix}
\]

b) Find the solution of the system

\[
\begin{align*}
x_1' &= x_1 - 2x_2 \\
x_2' &= x_1 + 3x_2
\end{align*}
\]

that satisfies the initial conditions \(x_1(0) = 1\) and \(x_2(0) = 1\). Write down the answer using real-valued functions.

Problem 7  Suppose that \(A\) is an \(m \times n\)-matrix with real entries. Prove that \(x \cdot A^T A x \geq 0\) for each \(x\) in \(\mathbb{R}^n\) and therefore each real eigenvalue of the matrix \(A^T A\) is non-negative.