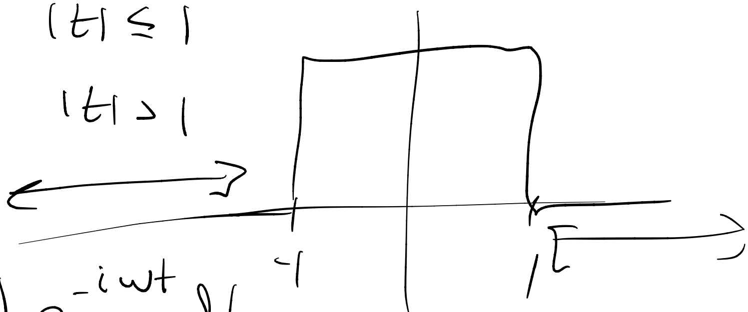
$$x(t) = \begin{cases} 1 & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$$



$$X(w) = \frac{1}{2\pi}\int_{-1}^{1} x(t)\, e^{-iwt}\, dt$$

$$= \frac{1}{2\pi}\int_{-1}^{1} 1\cdot e^{-iwt}\, dt = \frac{1}{2\pi}\left[\frac{e^{-iwt}}{-iw}\right]_{-1}^{1}$$

$$-\frac{1}{2\pi i w}\left(e^{-iw\cdot 1} - e^{-iw(-1)}\right)$$

$$\frac{1}{2\pi i w}\left(e^{iw} - e^{-iw}\right) = \frac{1}{\pi w}\left(\frac{e^{iw} - e^{-iw}}{2i}\right)$$

$$\frac{1}{\pi w}\left(\sin(w)\right)$$

$$\begin{array}{ccc} & x(t) & \\ \frac{1}{2\pi}\mathcal{F}^{-1}(X(w)) \uparrow & & \downarrow \frac{1}{2\pi}\mathcal{F}(x(t)) \\ & X(w) & \end{array}$$

$$x(t) \sim \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(s)\, e^{-i\omega s}\, ds\; e^{i\omega t}\, d\omega$$

3. $$x(t) = \begin{cases} a, & |t| \le 1/2a \\ 0, & \text{ellers} \end{cases}$$

$$\mathcal{L} = \int_0^{\infty} x\, e^{-st}\, dt$$

$$e^{-i\omega t}$$

$$X(\omega)\cdot H(\omega) = Y(\omega) \qquad \mathcal{F}\{x * y\} = X\, Y$$

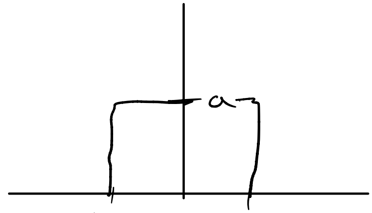
$$\mathcal{F}^{-1}\{Y(\omega)\} = y(t) \qquad x * y = \mathcal{F}^{-1}$$

$$X(\omega) = \frac{2a}{\omega}\sin\left(\frac{\omega}{2a}\right)$$

$$\lim_{a\to\infty} \frac{\sin\left(\frac{\omega}{2a}\right)}{\frac{\omega}{2a}} = \lim_{a\to\infty} \frac{\cos\left(\frac{\omega}{2a}\right)\left(-\frac{\omega}{2a^2}\right)}{-\frac{\omega}{2a^2}} = \lim_{a\to\infty} \cos\frac{\omega}{2a} = 1$$

3

$$f(t) = \begin{cases} a & |t| < 1/2a \\ 0 & \text{ellers} \end{cases}$$



$$\mathcal{F}(f) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$= \int_{-1/2a}^{1/2a} a\, e^{-i\omega t} dt = a\left(-\frac{1}{i\omega} e^{-i\omega t}\Big|_{-1/a2}^{1/2a}\right)$$

$$= -\frac{a}{i\omega}\left(e^{-i\omega/2a} - e^{i\omega/2a}\right)$$

$$= \frac{a}{\omega}\left(\frac{e^{i\omega/2a} - e^{-i\omega/2a}}{i}\right) = \frac{2a}{\omega} \sin \omega/2a$$

$$\lim_{a\to\infty} \frac{2a}{\omega} \sin \omega/2a = \lim_{a\to\infty} \frac{\sin \omega/2a}{\omega/2a}$$

$$\lim_{x\to\infty} f(x) = 0 = \lim_{x\to\infty} g(x)$$

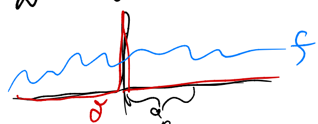
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$$

$$\lim_{a\to\infty} \frac{\sin\frac{\omega}{2a}}{\frac{\omega}{2a}} = \lim_{a\to\infty} \frac{\cos\left(\frac{\omega}{2a}\right)\cancel{\left(-\frac{\omega}{2a^2}\right)}}{\cancel{\left(-\frac{\omega}{2a^2}\right)}}$$

$$= \lim_{a\to\infty} \cos\left(\frac{\omega}{2a}\right) = \underline{\underline{1}}$$

$$x(t) \sim \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} x(s)e^{-i\omega s}\,ds\; e^{i\omega t}\,d\omega$$

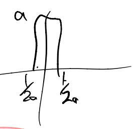
4) $\delta(x)$



$$\int_{-\infty}^{\infty}\delta(x)\,dx = 1 \qquad \int_{-\infty}^{\infty}\delta(x)f(x)\,dx = f(0)$$

$$\boxed{\int_{-\infty}^{\infty}\delta(x-a)f(x)\,dx = f(a)}$$

SVAR≈ $\frac{1}{2\pi}$



$$\mathcal{F}\{\delta\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(t)\underbrace{e^{-i\omega t}}_{f}\,dt = \frac{1}{2\pi}e^{-i\omega\cdot 0}$$

1

$$= \frac{1}{2\pi}$$

HINT TIL 6 OG 7:

$$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt = \int_{-\infty}^{\infty} f(t)e^{i\omega t}\,dt$$

BARE HVIS:

f(t) er jevn

$$\mathcal{F} = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)e^{-i\omega t}\,dt \qquad \mathcal{F}^{-1} = \int_{-\infty}^{\infty} X(\omega)e^{i\omega t}\,d\omega$$

$$\mathcal{F} \simeq \mathcal{F}^{-1}$$

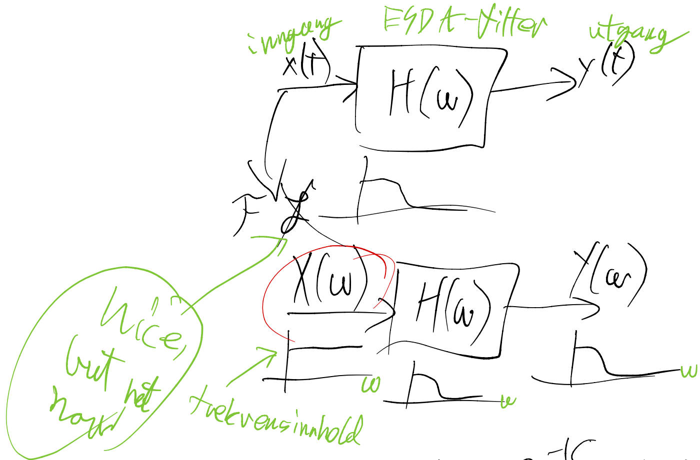
3: $\text{sinc}(a\omega)$

7: $\mathcal{F}\{\text{sinc}(x)\} \sim \mathcal{F}^{-1}\{\text{sinc}\}$

$$\mathcal{F}\{\text{boks}\} = \text{sinc}$$

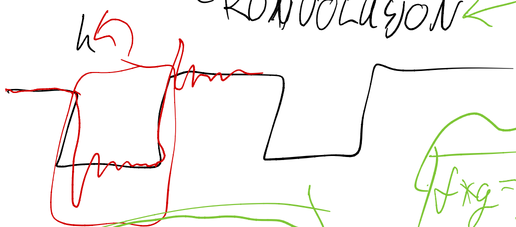
$$\mathcal{F}\{\text{sinc}\} = \mathcal{F}^{-1}\{\text{sinc}\} = \cancel{\mathcal{F}^{-1}}\{\cancel{\mathcal{F}}\{\text{boks}\}\} = \underline{\text{boks}}$$

$\mathcal{F}$ og $\mathcal{F}^{-1}$ samme, da kan vi bruke svaret fra oppg 3



$$x(t) * y(t) = \mathcal{F}^{-1}\{X(\omega)Y(\omega)\}$$

KONVOLUSJON ← NOE NYTT



$$f * g = \int_{-\infty}^{\infty} f(v)\,g(z-v)\,dv$$

x, h → y ?

Vi har

Vi vil ha

$$X(\omega)\cdot H(\omega) = Y(\omega)$$

$$x(t) * h(t) = y(t) = \mathcal{F}^{-1}\{\underbrace{X(\omega)H(\omega)}_{Y(\omega)}\}$$

Dette hadde vært lett

MEN DETTE ER ET ALTERNATIV