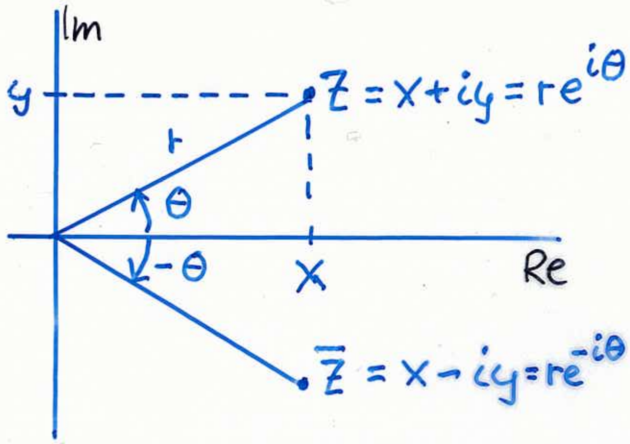
Komplekse tall (K 13.1, 13.2 + side 57-58)

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

normalform   polarform

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta \quad \text{(Eulers formel)}}$$



$x = \operatorname{Re} z$ $x = r\cos\theta$

$y = \operatorname{Im} z$ $y = r\sin\theta$ (HP30S: R◁▷P)

$r = |z|$ $r = \sqrt{x^2+y^2}$

$\theta = \arg z$ $\tan\theta = y/x$

$-\pi < \operatorname{Arg} z \leq \pi$

$\theta = \arctan\frac{y}{x}$ for z i 1. og 4. kvadrant $\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$

$\theta = \arctan\frac{y}{x} + \pi$ for z i 2. og 3. kvadrant $\left(\frac{\pi}{2} < \theta < \frac{3\pi}{2}\right)$

	normalform	polarform
$z_1 = z_2$	$x_1 = x_2$ og $y_1 = y_2$	$r_1 = r_2$ og $\theta_1 = \theta_2 + 2k\pi$ $(k = 0, \pm1, \pm2, \ldots)$
$\boxed{i^2 = -1}$ $z_1 \cdot z_2$	$(x_1 + iy_1)(x_2 + iy_2) = \ldots$	$r_1 r_2 e^{i(\theta_1 + \theta_2)}$
$\frac{z_1}{z_2}$	$\frac{z_1 \bar{z}_2}{z_2 \bar{z}_2} = \frac{z_1 \bar{z}_2}{\lvert z_2\rvert^2} = \ldots$	$\frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

n-te røtter: $\omega = \sqrt[n]{z}$

$$\omega^n = z$$

$$R^n e^{in\phi} = re^{i\theta}$$

$$R^n = r \quad \text{og} \quad n\phi = \theta + 2k\pi$$

$$R = \sqrt[n]{r} \quad \text{og} \quad \phi = \frac{\theta + 2k\pi}{n}$$

$$\boxed{\omega_k = \sqrt[n]{r}\, e^{i(\theta+2k\pi)/n} \quad , \quad k = 0, 1, 2, \ldots, n-1}$$

Komplekse løsninger i $y'' + ay' + by = 0$

$$e^{(\alpha+i\beta)x} = e^{\alpha x + i\beta x} = e^{\alpha x}(\cos\beta x + i\sin\beta x)$$

$$e^{(\alpha-i\beta)x} = e^{\alpha x - i\beta x} = e^{\alpha x}(\cos\beta x - i\sin\beta x)$$

Sum: $2e^{\alpha x}\cos\beta x$

Diff.: $2i\,e^{\alpha x}\sin\beta x$

$$\boxed{\begin{array}{c} e^{\alpha x}\cos\beta x \\ e^{\alpha x}\sin\beta x \end{array}}$$ Reelle løsninger