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EXAM IN TMA4110/15 MATEMATIKK 3

English

August 17, 2011

Time: 9-13

Examination aids (code C): Simple calculator (HP30S or Citizen SR-270X)

Rottman: *Collection of Mathematical Formulas*

Sensur: September 7, 2011

Full reasoning should be given for all answers so that it is clear how the answer was obtained.

Problem 1 Find all complex numbers s such that $\operatorname{Im}(-z + i) = (z + i)^2$. Draw the solutions on a diagram.

Problem 2

a) Solve the initial value problem:

$$y'' - 7y' + 12y = 0, \quad y(0) = 2, \quad y'(0) = 1.$$

b) Find the general solution to the differential equation:

$$y'' - 7y' + 12y = 50 \cos 3x - 2xe^{3x}.$$

Problem 3 Consider the differential equation

$$xy'' - y' + 4x^3y = 0.$$

a) Show that $y_1(x) = \sin(x^2)$ is a solution.

b) Find another solution $y_2(x)$ such that y_1 and y_2 are linearly independent.

Problem 4 Let

$$A = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}.$$

a) Find a basis for the null space, $\text{Null}(A)$, and a basis for the row space, $\text{Row}(A)$.

b) Find a basis for the column space, $\text{Col}(A)$. What is $\text{rank}(A)$?

Problem 5 Let

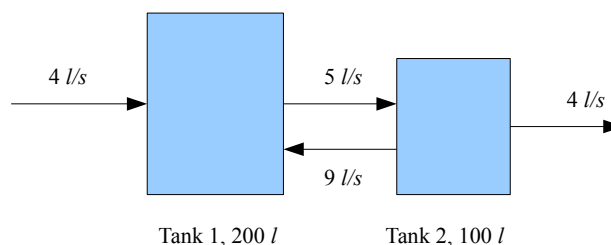
$$A = \begin{bmatrix} 1 & 2 & a \\ 3 & 7 & b \\ 2 & 9 & c \end{bmatrix}.$$

a) For which numbers a , b , and c is A an invertible matrix?

b) Find numbers a , b , and c such that A^{-1} is an integer matrix.

Problem 6

The diagram shows two tanks. Both contain salt water; tank 1 holds 200 litres and tank 2 holds 100 litres. Pure water enters tank 1 at a rate of 4 litres per second. The flow between the tanks, and out of tank 2, is shown in the figure. The tanks are stirred so that the salt in each is evenly distributed.



a) Find a system of differential equations which describes the salt concentrations $y_1(t)$ and $y_2(t)$ of, respectively, tank 1 and tank 2.

b) Determine $y_1(t)$ and $y_2(t)$ given that at the start ($t = 0$), tank 1 contained 100 grammes of salt and tank 2 contained 200 grammes of salt.

Problem 7 Let A and B be square ($n \times n$)–matrices. Show that AB is invertible if and only if both A and B are invertible.