NTNU
Norwegian University of Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4110/TMA4115 Calculus 3**

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Examination date: 09. August 2019

Examination time (from–to): 09:00–13:00

Permitted examination support material: C: Specified printed and hand-written support material is allowed. A specific basic calculator is allowed.

Other information:
The exam consists of 10 subproblems. All subproblems are given equal weight. Give reasons for all answers. This year we specify that NO printed or handwritten support material is allowed.

Language: English

Number of pages: 2

Number of pages enclosed: 0

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Informasjon om trykking av eksamensoppgave	
Originalen er:	
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skal ha flervalgskjema ☐	

Date Signature

Problem 1

a) Find all solutions of

$$z^5 = i$$

in $\mathbb{C}$ and sketch them in the complex plane.

b) Let z og w be complex numbers. Show that

$$\overline{z/w} = \overline{z}/\overline{w}.$$

Problem 2

a) Let A be a real valued $m \times n$-matrix. Give the definition of the null space of A. Show that the null space is a subspace of $\mathbb{R}^n$.

Consider the matrix

$$A = \begin{bmatrix} 2 & 4 & 0 \\ -5 & -4 & 6 \\ 1 & -2 & -4 \end{bmatrix}.$$

b) Is

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

in the null space of A?

Find a basis for Col$A$ and a basis for Null$A$. Decide the dimension of these subspaces.

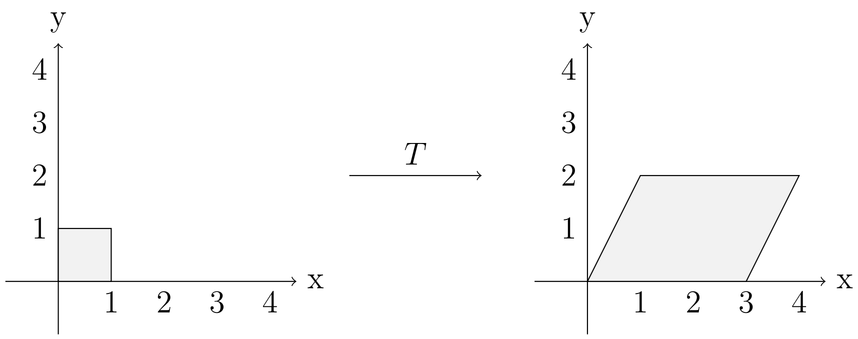
c) Find an orthogonal basis for Col$A$. Compute the orthogonal projection of $\begin{bmatrix} 3 \\ 3 \\ 9 \end{bmatrix}$ on Col$A$.

Problem 3 A linear transform $T : \mathbb{R}^2 \to \mathbb{R}^2$ maps the square with corners located in

$$(0,0),\ (1,0),\ (0,1) \text{ and } (1,1)$$

to the parallelogram spanned by

$$\begin{bmatrix} 3 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$



Find the standard matrix $[T]$ of T and compute $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$. Find $\ker T$. Is T surjective?

Problem 4 Find the general solution to the system of differential equations

$$\mathbf{y}' = A\mathbf{y} \qquad \text{where} \qquad A = \begin{bmatrix} 3 & -4 \\ 2 & -3 \end{bmatrix}.$$

Problem 5 Find an explicit formula for A^n when $n \geq 0$ and

$$A = \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}.$$

Problem 6 Give the definition of a diagonalizable square matrix. Show that a diagonalizable 2×2-matrix with an eigenvalue of multiplicity two must be diagonal.

Problem 7 Let $\mathbf{v}, \mathbf{w}$ be vectors in $\mathbb{R}^n$, where $\mathbf{v} \neq 0$. Define the projection $P_{\mathbf{v}}(\mathbf{w})$ of $\mathbf{w}$ on $\mathbf{v}$. Show that the projection $P_{\mathbf{v}} : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation. Show that $P_{\mathbf{v}}(\mathbf{w})$ and $\mathbf{w} - P_{\mathbf{v}}(\mathbf{w})$ are orthogonal.