Examination paper for **TMA4110/TMA4115 Calculus 3**

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**Examination date:** 09. August 2019  
**Examination time (from–to):** 09:00–13:00  
**Permitted examination support material:** C: Specified printed and hand-written support material is allowed. A specific basic calculator is allowed.

**Other information:**  
The exam consists of 10 subproblems. All subproblems are given equal weight. Give reasons for all answers. This year we specify that NO printed or handwritten support material is allowed.

**Language:** English  
**Number of pages:** 2  
**Number of pages enclosed:** 0

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**Informasjon om trykking av eksamensoppgave**  
**Originalen er:**  
1-sidig ☐  2-sidig ☒  
sort/hvit ☒  farger ☐  
skal ha flervalgskjema ☐

**Checked by:**

__________________________________________
Date  Signature
Problem 1

a) Find all solutions of

\[ z^5 = i \]

in \( \mathbb{C} \) and sketch them in the complex plane.

b) Let \( z \) og \( w \) be complex numbers. Show that

\[ \frac{\bar{z}}{w} = \frac{\bar{z}}{\bar{w}}. \]

Problem 2

a) Let \( A \) be a real valued \( m \times n \)-matrix. Give the definition of the null space of \( A \). Show that the null space is a subspace of \( \mathbb{R}^n \).

Consider the matrix

\[
A = \begin{bmatrix}
2 & 4 & 0 \\
-5 & -4 & 6 \\
1 & -2 & -4
\end{bmatrix}.
\]

b) Is

\[
\begin{bmatrix}
1 \\
2 \\
-1
\end{bmatrix}
\quad \text{or} \quad
\begin{bmatrix}
2 \\
-1 \\
1
\end{bmatrix}
\]

in the null space of \( A \)?

Find a basis for \( \text{Col}A \) and a basis for \( \text{Null}A \). Decide the dimension of these subspaces.

c) Find an orthogonal basis for \( \text{Col}A \). Compute the orthogonal projection of

\[
\begin{bmatrix}
3 \\
3 \\
9
\end{bmatrix}
\]
on \( \text{Col}A \).
Problem 3  A linear transform $T : \mathbb{R}^2 \to \mathbb{R}^2$ maps the square with corners located in

$$(0, 0), (1, 0), (0, 1) \text{ and } (1, 1)$$

to the parallelogram spanned by

$$\begin{bmatrix} 3 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$