NTNU
Norwegian University of Science and Technology

Department of Mathematical Sciences

Examination paper for **TMA4115 Calculus 3**

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Examination date: May 31 2019

Examination time (from–to): 09:00–13:00

Permitted examination support material: C: Specified printed and hand-written support material is allowed. A specific basic calculator is allowed.

Other information:
The exam consists of 10 subproblems. All subproblems are given equal weight. Give reasons for all answers. This year we specify that NO printed or handwritten support material is allowed.

Language: English

Number of pages: 2

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave
Originalen er:
1-sidig ☐ **2-sidig** ☒
sort/hvit ☒ **farger** ☐
skal ha flervalgskjema ☐

Date Signature

Problem 1

a) Find bases for the column space and the null space of the matrix $\begin{bmatrix} 1 & -1 & 5 \\ -2 & 2 & -10 \end{bmatrix}$.

b) Find all complex solutions of the system

$$\begin{aligned} x - y + iz &= 0 \\ -x + iy - z &= 0. \end{aligned}$$

Problem 2

a) Find a polynomial $f(x) = ax^2 + bx + c$ containing the points $(-1, 5)$, $(0, 1)$, $(1, -1)$ and $(2, -1)$ in $\mathbb{R}^2$.

Use the method of least squares to find the straight line $g(x) = dx + e$ which is the best fit to the four points.

Sketch the graphs of f and g.

b) Let

$$A = \begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ -1 \\ -1 \end{bmatrix}.$$

Find the orthogonal projection of $\mathbf{b}$ onto $\mathrm{Col}A$.

Is $P_{\mathrm{Col}A} : \mathbb{R}^4 \to \mathbb{R}^4$, the linear transformation projecting onto $\mathrm{Col}A$, injective?

Problem 3 Write the definition of linear independence, and use it to show that the vectors $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 6 \\ 0 \end{bmatrix} \right\}$ are lineary independent.

Problem 4 Find a system of linear differential equations

$$\mathbf{y}'(t) = A\mathbf{y}(t)$$

having the general solution

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} + c_1 \begin{bmatrix} 3 \\ -1 \end{bmatrix} e^{3t}$$

and sketch the phase plane of the system.

Problem 5 A course in linear algebra is being lectured in two parallels, one in the room S7 and one in S8. Both lecturers are very poor, and therefore the students change frequently between the parallels. The probability that a student changes parallel after any given week of lectures is 40%.

Determine a stochastic matrix M that describes this process. At the beginning of the semester, there are respectively 180 and 140 students assigned to the parallels. How will the students be distributed after 14 weeks of lecturing, given that the students actually still continue to attend the lectures.

Problem 6 Let $p(x) = 3x^2 - 3x - 6$, $q(x) = x^2 - x - 8$ og $r(x) = 4x^2 - 9x + 3$. Determine if $r(x)$ can be writen as a linear combination of $p(x)$ and $q(x)$.

Determine if p and q are orthogonal with respect to the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\,dx.$$

Problem 7 A projection matrix M is a matrix such that $M^2 = M$.

a) Let $T\colon \mathbb{R}^m \to \mathbb{R}^m$ be the function given by

$$T(x_1, x_2, \ldots, x_m) = (x_1, x_2, 0, \ldots, 0),$$

where $m \geq 2$. Show that T is a linear transformation and find the standard matrix $[T]$. Show that $[T]$ is a projection matrix, and find the eigenvalues of $[T]$ for all $m \geq 2$.

b) Let M be a projection matrix which is not the zero matrix or the identity matrix. Show that M is not invertible. Show that M has eigenvalues 0 and 1, and no other eigenvalues.